1 Preliminaries

- Objectives
  1. To measure the focal length of a lens.
  2. To study the concepts involved in image formation.
  3. To measure the index of refraction of glass via the lens maker’s equation.

- Safety
  There is no particular danger associated with this lab. Use caution in handling the lenses and ground glass plates— they may have sharp edges. Be careful not to break the glasses, and please handle them by their edges to avoid leaving oils from your fingers on the surfaces.

- Equipment
  Light source and aperture (object), optical rail, lens, ground-glass screen, lens and screen holders, spherometer, ruler, caliper.

2 Theory

Light can be described as an electromagnetic wave—that is, propagating electric and magnetic fields. Visible light has a wavelength in the range of approximately 400 to 700 nm. The wave nature of light becomes necessary to explain the behavior of light when it interacts with objects that have features on the same size scale as a wavelength. (We will perform two impressive experiments later to investigate the “wave nature” of light.)

If light only interacts with objects that are very large compared to its wavelength, the wave nature of light may be difficult to observe. In this regime, light is commonly treated as rays that travel in straight line paths. This realm of optics is called Geometrical Optics, where simple geometry is used to calculate how light rays are reflected or transmitted at interfaces between two different materials.

2.1 Positive Focal Length Lens

A lens is an optical instrument which can redirect light rays that are incident upon it. The focal length of a lens, \( f \), indicates where a bundle of parallel light rays entering one side of the lens will be focused on the opposite side (if \( f \) is positive). Figure 1a depicts a lens with a positive focal length (we will not consider negative focal length lenses in this experiment).

The focal length of a lens is determined by only two factors: the lens material, and the curvature of the lens surfaces. For a thin lens in air, the focal length can be found according to the lens maker’s equation,

\[
\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right),
\]
where $n$ is the index of refraction of the lens material, $R_1$ is the radius of curvature of the first surface the light encounters, and $R_2$ is the radius of curvature of the second surface. The index of refraction is the factor by which the speed of light in matter is reduced compared to its speed in a vacuum. Figure 1b depicts the radii of curvature, and the caption explains the sign convention for $R$.

**Figure 1:** a) A lens with a positive focal length brings parallel rays of light from the left to a focus on the right. b) A lens made of two spherical surfaces. Note the sign convention: if the center of the spherical surface is on the side from where light has come, the radius of curvature is negative. If the center of the sphere is on the side of the lens to where light is going, the radius of curvature of that surface is positive.

### 2.2 Imaging

A positive focal length lens is capable of projecting an image of a real object on a screen, when the object and screen are at particular locations relative to the lens. Consider an object located a distance $o$ to the left of a thin lens of focal length $f$. An image is formed a distance $i$ to the right of the lens, according to the equation,

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}.$$  \hspace{1cm} (2)

Light will scatter off of an object in all directions, and the lens will redirect the rays that impinge upon it. Figure 2 follows three particular rays that are scattered from one point on an object (the top of a matchstick) as they travel through the lens and converge to a point. The image of the tip of the matchstick is formed at the point where the rays converge. Indeed, an image of the entire matchstick is formed at location $i$ but to avoid cluttering the figure, we have neglected to draw rays from each point on the matchstick.

**Figure 2:** Three rays are traced from a point on the object through the lens to the point where they converge. The focal points on either side of the lens are labeled $f$. 

2
Let’s take a closer look at the three rays in Figure 2. The first ray is traced from the object point in question parallel to an axis through the center of the lens. Like in Figure 1a, this ray will pass through the focal point on the opposite side of the lens. By the same logic, a ray leaving the object and passing through the front focal point will emerge from the lens parallel to the horizontal axis. Lastly, a ray passing through the center of a thin lens is undeviated. Tracing these three rays through the lens allows one to find the image location $i$. (Remember: even though only three rays are depicted, the image location is where all rays emanating from the object and passing through the lens would converge.)

Notice that the image may be magnified with respect to the object. If the object height is $h_o$ and the image height is $h_i$, then the magnification is defined as

$$m = -\frac{h_i}{h_o}. \quad (3)$$

The negative sign indicates that the image is inverted compared to the object. Using simple trigonometry, the magnification can be written in terms of the object and image distances instead:

$$m = -\frac{i}{o}. \quad (4)$$

3 Procedure

3.1 Measuring the focal length, and calculating the index of refraction

Record the number of your lens here. Lens number: 

1. Prove in the space that follows that Eq. 2 can be rewritten as $oi = f(o + i)$, and call it Eq. 5.

2. Use the optical rail, the object, lens, and ground glass plate to measure at least six distinct object and image distances $(o, i)$. Of course, you are restricted in your choice of $o$ such that all components can still be mounted on the rail. (Values of $o$ in the range of 10-25 cm usually work satisfactorily.) Measure $o$ and $i$ using the ruler that is integrated into the optical rail. Be sure to correct for the offsets between the number you record from the ruler and the actual location of the component. Set up a spreadsheet to contain all of your recorded values. Estimate the uncertainty in your $o$ and $i$ measurements.

*Hint:* You may experience some difficulty in finding the precise location where the image is in focus. Recognize first that if you swap object and image distances, while experimentally different, they will produce the same point on a plot of $oi$ vs. $o + i$. (For instance, if an image is formed with $o = 15$ cm and $i = 20$ cm, then an image must also be formed when $o = 20$ cm and $i = 15$ cm, according to Eq. 2. Yet these distinct measurements produce the same $oi$ and $o + i$ values.)

*Here’s the hint:* Notice that the image location is much easier to pinpoint if the object distance exceeds the image distance. The tradeoff is a lower image brightness. (Can you think of why?)

3
3. Plot $o_i$ vs. $o + i$. If you do not have six well-spaced points, make additional measurements. Fit a straight line to the data and analyze the fit with the Linest function in Excel. In the space below, report the focal length (and uncertainty) for your lens. Discuss the value of the intercept.

4. Find the index of refraction of the glass from which your lens is made. See the appendix for details on the spherometer. Use the spherometer to find the radius of curvature of each surface of your lens.
   
   (a) You will need to find $a$ from Figure A.2b using the calipers. Measure $a$ from the center leg to each outer leg and use the average of the three in your calculations.
   
   (b) Measure $b$ with the spherometer. Set the zero position accurately using a flat glass plate.
   
   (c) Use Eq. A.5 and Maple to calculate the radii of curvature for your lens, complete with uncertainties.
   
   (d) Now armed with the radii of curvature, and the experimental value of the focal length, use the lens maker’s equation (Eq. 1) and Maple to calculate the index of refraction $n$ and its uncertainty.

3.2 Concepts in image formation

Use one of your $(o,i)$ pairs to find an image on the ground glass screen.

1. Measure the object and image height with a ruler, and compare the magnifications found using Eqs. 3 and 4.

2. Predict and record in the space below what you believe will happen to the image on the screen if you block the bottom half of the lens with an opaque card.
3. Block the bottom half of the lens with an opaque card and record what effect is produced in the image. *Explain* the result you observe and reconcile any differences between your predictions and observations.

4. Now remove the ground glass screen and substitute your eye, located at a distance somewhat greater than $i$. Look directly into the lens and practice positioning your eye until you see a clear image of the object centered in the lens. Predict and record here what you expect will happen if your partner blocks the bottom half of the lens with an opaque card.

5. Have your partner block the bottom half of the lens with the card, and record what you see with your eye. Explain your observations, and reconcile any differences with your earlier predictions.

4  Hand in:

- Spreadsheet and plot,
- Maple analysis of the radii of curvature and index of refraction calculations, and
- Lab handout with properly written answers and observations.
Appendix – The Spherometer

The radius of curvature $R$ of a spherical surface can be measured with a device called a spherometer, as shown in Figure A.1. The spherometer measures, on a spherical surface, how far a point is above the midpoint of an equilateral triangle itself constructed from three other points on the same surface. These concepts are illustrated in Figure A.2.

**Figure A.1:** The spherometer, used to measure the radius of curvature of a spherical surface.

**Figure A.2:** Spherometer diagrams as viewed from a) the side, and b) top down. The outer legs of the spherometer are labeled $L_1$, $L_2$, and $L_3$, and the middle leg is $M$. The distance between $M$ and any outer leg is $a$. The height difference between the middle leg and all outer legs is $b$. The radius of curvature of the surface being measured is $R$. 
The equation of a spherical surface, in a Cartesian coordinate system whose origin is at the center of the sphere, is

\[ x^2 + y^2 + z^2 = R^2. \]  \hfill (A.1)

The equation of the same surface in a second Cartesian coordinate system whose origin is a distance \( z_0 \) above the first, along the common \( z \)-direction is

\[ x^2 + y^2 + (z + z_0)^2 = R^2. \]  \hfill (A.2)

Let the \( xy \) plane of this second system (for which \( z = 0 \)) be the plane formed by the tips of the three outer legs of the spherometer, \( L_1, L_2, \) and \( L_3 \). As depicted in Figure A.2b, this \( xy \) plane intersects the spherical surface, and therefore

\[ x^2 + y^2 = a^2. \]  \hfill (A.3)

Combining Eqs. A.2 and A.3, and remembering that \( z = 0 \) here, we have

\[ a^2 + z_0^2 = R^2. \]  \hfill (A.4)

Substituting \( z_0 = R - b \) into Eq. A.4 and solving for \( R \) yields

\[ R = \frac{a^2 + b^2}{2b}, \]  \hfill (A.5)

which allows one to calculate the radius of curvature of the surface after having measured \( a \) and \( b \).