[24 points] A point particle with charge $7 \mu C$ located at $(x, y) = (0, 0)$ m interacts with another point particle with charge $-4 \mu C$ located at $(x, y) = (0.2, 0.4)$ m as shown. The mass of each particle is 0.01 kg.

(a) Calculate the electric potential due to the $7 \mu C$ charge at the location of the $-4 \mu C$ charge (taking the convention $V = 0$ at "infinity" as usual).

$$ V = k \frac{(7 \mu C)}{\sqrt{(x)^2 + (y)^2}} $$

**Answer:** Electric Potential = \(141,000\) (units)

(b) Calculate the potential energy of the configuration.

$$ U = (7 \mu C) V = -0.563 \text{ J} $$

**Answer:** Potential Energy = \(-0.563\) (units)

(c) If the $-4 \mu C$ charge is released from rest at the position $(x, y) = (0.2, 0.4)$ m and the $7 \mu C$ charge remains fixed at the origin, find the speed of the $-4 \mu C$ particle after it has moved a distance of 0.1 m.

$$ \Delta U = k \frac{(-4 \mu C)(7 \mu C)}{\sqrt{(x)^2 + (y)^2}} - (-0.563 \text{ J}) = 0.162 \text{ J} $$

$$ \frac{1}{2} \mu v^2 = 0.162 \text{ J} $$

$$ \Rightarrow v = 5.69 \text{ m/s} $$

**Answer:** Speed = \(5.69\) (units)
2 [21 points] Point charge \( q_1 \) is located at position \((x, y) = (0, 0)\) and point charge \( q_2 \) is located at position \((x, y) = (a, 0)\).

(a) Give the components of a unit vector pointing from \((0, 0)\) to \((a, b)\). Your answer should involve \(a\) and \(b\).

\[
\frac{a \hat{\imath} + b \hat{j}}{\sqrt{a^2 + b^2}}
\]

Answer:
\[
\frac{a}{\sqrt{a^2 + b^2}} \hat{\imath} + \frac{b}{\sqrt{a^2 + b^2}} \hat{j}
\]

(b) Calculate the electric field \( \vec{E} \) due to \( q_1 \) and \( q_2 \) at the point \( P \) with coordinates \((x, y) = (a, b)\). Your answer should involve \(a\) and \(b\).

\[
\vec{E} = \frac{k q_2}{b^2} \hat{j} + \frac{k q_1}{(\sqrt{a^2 + b^2})^3} \left( \frac{a \hat{\imath} + b \hat{j}}{\sqrt{a^2 + b^2}} \right)
\]

Answer:
\[
\vec{E} = \frac{k q_1}{(a^2 + b^2)^{3/2}} \hat{\imath} + \frac{k q_2}{b^2 (a^2 + b^2)^{3/2}} \hat{j}
\]

3 [15 points] A point charge \( q \) is located inside a cube-shaped imaginary box, but it is not necessarily at the center. There are no other charges or materials affecting this location.

(a) What is the largest that the flux through any one face of the box could be if you are allowed to move the charge anywhere within the box?

Answer: \( \frac{q}{2\epsilon_0} \)

(b) What is the largest that the flux through the entire box could be if you are allowed to move the charge anywhere within the box, but not outside?

This is Gauss's Law. It doesn't matter where the charge is.

Answer: \( \frac{q}{\epsilon_0} \)

(c) If the charge is placed in a corner of the cube, what is the flux through one of the sides that is opposite the corner (that does not touch the corner)?

\( \frac{1}{8} \) of the possible flux goes into the volume of the cube. \( \frac{1}{3} \) of enters each opposite face.

Answer: \( \frac{q}{24\epsilon_0} \)
4 [27 points] A solid sphere of radius 4 cm is composed of nonconducting material with a uniform charge density of \( \rho = 3 \, \mu C/m^3 \). It is surrounded by a concentric uncharged conducting spherical shell of inner radius 6 cm and outer radius 8 cm.

(a) Find the magnitude of the electric field at a distance of 5 cm from the center of the spheres.

\[
\oint E \cdot dA = \frac{9\pi}{\varepsilon_0}
\]

\[
E \Delta A (0.05m)^2 = \frac{\rho \times \frac{4}{3} \pi (0.04m)^3}{\varepsilon_0}
\]

\[\rightarrow E = 2891 \frac{N}{C}\]

Answer: \( E = \frac{2890}{\text{units}} \frac{N}{C} \)

(b) Find the magnitude of the force on a 0.001 C charge placed at a distance of 5 cm from the center of the spheres.

Answer: \( F = \frac{2.89}{\text{units}} \frac{N}{C} \)

(c) Find the magnitude of the electric field at a distance of 7 cm from the center of the spheres.

This is in the conductor.

\[ E = 0 \]

Answer: \( E = \frac{0}{\text{units}} \frac{N}{C} \)

(d) How much total charge is on the inner surface of the conductor, at 6 cm from the center of the spheres?

It is the opposite of how much is in the \( \rho \) region; \( \frac{4}{3} \pi (0.04)^3 = 8.04 \times 10^{-10} \, C \)

Answer: \( -8.04 \times 10^{-10} \, C \)

(e) How much total charge is on the outer surface of the conductor, at 8 cm from the center of the spheres?

Conductor is uncharged, so this is the opposite of part (d).

Answer: \( 8.04 \times 10^{-10} \, C \)
[20 points] 4 μC of charge is distributed uniformly on the x-axis between x = 0 cm and x = 3 cm. Find the components of the electric field at the position (x, y) = (0 cm, 2 cm).

\[ E = \int \frac{k dq}{r^2} \]

\[ E_x = -k \int_0^{0.03 m} \frac{x dx}{\sqrt{x^2 + (0.02 m)^2}} \]

\[ dq = \frac{4 \mu C}{0.03 m} \frac{dx}{x} \]

\[ r = \sqrt{x^2 + (0.02 m)^2} \]

\[ \hat{r} = \frac{-x \hat{i} + 0.02 m \hat{j}}{\sqrt{x^2 + (0.02 m)^2}} \]

\[ \Rightarrow k \lambda \left[ \frac{1}{\sqrt{0.0013}} - \frac{1}{\sqrt{0.0004}} \right] \]

\[ \approx -2.667 \times 10^7 \frac{N}{C} \]

Answer: \( E_x = -2.67 \times 10^7 \frac{N}{C} \) (units)