The exam will cover Chapters 32, 34, 35 and 36. The equations and tables that will appear on the exam are on the next page. The big ideas that we discussed were:

**Chapter 32:**
- A current in a wire loop or circuit generates a magnetic flux through that same loop. The flux is proportional to the current, and the constant of proportionality is the self inductance, \( L \): \( \phi = LI \)
- The inductance of a circuit or loop only depends on its geometry, not on its current.
- Faraday’s law then says that self-induced emf is \( |\mathcal{E}| = -L \frac{dI}{dt} \). The faster current tries to change, the more the induced emf will push back and try to keep it from changing. (Lenz’s law)
- In an RL circuit the outcome for how the current actually changes with time if a battery of emf \( \mathcal{E} \) is suddenly connected in series with \( R \) and \( L \) might seem complicated. The battery is pushing in one direction, but the self induced emf pushes back. In class we set up a differential equation and found the solution \( I = (\mathcal{E} / R)(1 - e^{-t/\tau}) \) where the time constant \( \tau = L / R \). If it comes to equilibrium and then the battery is suddenly replaced by a wire, the current dies out according to \( I = (\mathcal{E} / R)e^{-t/\tau} \).
- The energy stored in an inductor \( L \) carrying current \( I \) is \( U = \frac{1}{2} LI^2 \) (which can be used to do work later since the current “wants” to avoid slowing down).
- The energy density in a region of space containing a magnetic field strength \( B \) is \( u_B = \frac{B^2}{2\mu_0} \)

**Chapter 34:**
- Maxwell-Ampere law: We find that \( \oint \vec{B} \cdot d\vec{l} \) around a loop depends on both the current flowing through the loop and the changing electric flux through the loop. To calculate these two quantities we need to choose a surface bounded by the loop. For some surfaces, there will be more current and less flux, and for others we find the opposite. But regardless of which surface you choose the sum \( \mu_0 I_{\text{enclosed}} + \mu_0 \varepsilon_0 \frac{d\phi}{dt} \) will remain the same. So choose a convenient surface!
- Practically speaking, parallel plate capacitor problems are the only ones that might utilize both terms in Maxwell-Ampere that are easy enough to do. One type of different problem is included on this worksheet.
- The other big idea is that Maxwell’s equations have wave solutions. The following are all important concepts and affect the mathematical form of the plane wave: the direction a plane wave moves in; its speed \( v \) and the relations between \( v, k, \lambda, f, \) and \( \omega \); the relation of \( E_{\text{max}} \) and \( B_{\text{max}} \); the relations of the directions of \( \vec{E}, \vec{B} \) and the direction of motion. Know how to get these from the mathematical form, and vice versa.
- We did not discuss 34.4-34.6.

**Chapter 35:**
- Geometric optics uses the ray approximation, which is valid when light reflects or refracts only from surfaces that are smooth at the scale of the wavelength.
- \( n = c / v \), Snell’s law (and total internal reflection) and the law of reflection are some major results.
- The fact that in most media, the index of refraction varies with wavelength is called “dispersion”. You should understand how a prism breaks white light into a rainbow of colors.
- In this chapter the basic laws are easy. It is the geometry that might confuse you.
Chapter 36:

- Images formed by spherical mirrors, refraction by a flat surface, and by thin lenses are the major ideas. You should be able to combine these if you have 2 lenses, or a lens and a mirror.
- Be able to use ray tracing and the formulas to find image locations and magnifications, real or virtual, upright or inverted.
- The sign conventions will be summarized in tables. Know how to use them.
- There are several other ideas in the chapter that were not emphasized much by me. Use your judgment when studying.

Equations

\[ c = 1/\sqrt{\mu_0 \varepsilon_0} \]
\[ E = Bc \]
\[ \vec{E} \times \vec{B} = \dot{\vec{c}} \]
\[ \dot{E} = E_{\text{max}} \cos(kx - \omega t) \hat{j} \]
\[ \dot{B} = B_{\text{max}} \cos(kx - \omega t) \hat{k} \]
\[ \lambda f = c \]
\[ k = 2\pi / \lambda \]
\[ v = \omega / k \]
\[ n = \frac{c}{v} \]
\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]
\[ M = \frac{h'}{h} = -\frac{q}{p} \]
\[ \frac{1}{p} + \frac{1}{q} = \frac{1}{f} = 2 \frac{R}{p} \]
\[ \frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \]
\[ \frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]
\[ \Phi = LI \]
\[ \mathcal{E}_L = -L \frac{dl}{dt} \]
\[ L = \mu_0 \frac{N^2}{\ell} A \]
\[ I = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau}) \]
\[ I = \frac{\mathcal{E}}{R} e^{-t/\tau} \]
\[ \tau = \frac{L}{R} \text{ (time to 63\%)} \]
\[ U = \frac{1}{2} LI^2 \]
\[ u_B = \frac{B^2}{2 \mu_0} \]

**Sign Conventions for Mirrors**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Positive When . . .</th>
<th>Negative When . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>Object location (p)</td>
<td>object is in front of mirror (real object).</td>
<td>object is in back of mirror (virtual object).</td>
</tr>
<tr>
<td>Image location (q)</td>
<td>image is in front of mirror (real image).</td>
<td>image is in back of mirror (virtual image).</td>
</tr>
<tr>
<td>Image height (h')</td>
<td>image is upright.</td>
<td>image is inverted.</td>
</tr>
<tr>
<td>Focal length (f) and radius (R)</td>
<td>mirror is concave.</td>
<td>mirror is convex.</td>
</tr>
<tr>
<td>Magnification (M)</td>
<td>image is upright.</td>
<td>image is inverted.</td>
</tr>
</tbody>
</table>

**Sign Conventions for Refracting Surfaces**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Positive When . . .</th>
<th>Negative When . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>Object location (p)</td>
<td>object is in front of surface (real object).</td>
<td>object is in back of surface (virtual object).</td>
</tr>
<tr>
<td>Image location (q)</td>
<td>image is in back of surface (real image).</td>
<td>image is in front of surface (virtual image).</td>
</tr>
<tr>
<td>Image height (h')</td>
<td>image is upright.</td>
<td>image is inverted.</td>
</tr>
<tr>
<td>Radius (R)</td>
<td>center of curvature is in back of surface.</td>
<td>center of curvature is in front of surface.</td>
</tr>
</tbody>
</table>

**Sign Conventions for Thin Lenses**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Positive When . . .</th>
<th>Negative When . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>Object location (p)</td>
<td>object is in front of lens (real object).</td>
<td>object is in back of lens (virtual object).</td>
</tr>
<tr>
<td>Image location (q)</td>
<td>image is in back of lens (real image).</td>
<td>image is in front of lens (virtual image).</td>
</tr>
<tr>
<td>Image height (h')</td>
<td>image is upright.</td>
<td>image is inverted.</td>
</tr>
<tr>
<td>R_1 and R_2</td>
<td>center of curvature is in back of lens.</td>
<td>center of curvature is in front of lens.</td>
</tr>
<tr>
<td>Focal length (f)</td>
<td>a converging lens.</td>
<td>a diverging lens.</td>
</tr>
</tbody>
</table>
A plane wave is traveling through a piece of glass. The magnetic field at position $(x, y, z)$ and at time $t$ is given by $\vec{B} = B_{\text{max}} \sin(ky + \omega t) \hat{i}$, where $B_{\text{max}} = 1.23 \times 10^{-6}$ T, $\omega = 6.54 \times 10^{9}$ s$^{-1}$, and $k = 30.7$ m$^{-1}$.

(a) Write down the correct similar expression for the electric field, $\vec{E}$, at position $(x, y, z)$ and at time $t$.

(b) The wave is traveling in the __________ direction and has speed _______________________ m/s.

(c) The index of refraction of the glass is ___________.

(d) The wavelength of the wave in the glass is ________________.

(e) The frequency of the wave is ________________ Hz.

A magnetic field varies in space near the $xyz$ axes according to $\vec{B} = 3x^2 \hat{k}$ where $B$ is in Tesla and $x$ is in meters. Find the amount of energy stored in the magnetic field inside the box defined by $0 < x < 0.5$ m, $0 < y < 0.5$ m and $0 < z < 0.5$ m.
A triangular glass prism with apex angle $\Phi = 50^\circ$ has an index of refraction $n = 2$. (a) If a light ray hits the left side at an angle $\theta_1 = 50^\circ$, at what angle relative to the normal will it exit the far side?

(b) Given that the index of refraction is actually a function of wavelength, not merely a constant $n = 2$, will the exit angle relative to the normal in (a) be greater or less for red light compared to blue light?

Another prism $(n = 1.3)$ has a square cross section, and a light ray is incident on the center of one side.

(a) What angle of incidence $\theta_1$ will cause the light ray to hit the upper right corner after entering the prism?

(b) What is the largest angle of incidence $\theta_1$ for which no light will emerge from the top side?
The figure shows a partially submerged person and a fish in water.

(a) Does the person see the fish in the general region of point \( a \) or point \( b \)?

(b) Does the fish see the person’s eyes in the general region of point \( c \) or point \( d \)?

A convex mirror has a radius of curvature of 40 cm. A 10-cm-tall object is placed 30 cm from the mirror.

(a) The image is _________ cm from the mirror.

(b) The image is _________ in front of _________ behind the mirror. (Circle the correct underlined response.)

(c) The image is _________ real _________ virtual.

(d) The image height is _______________ cm.

(e) The image is _________ upright _________ inverted.

(f) If the object has a speed 3 m/s at the instant it is 30 cm from the center of the mirror, the apparent speed of the image at this instant is _______________ m/s.
The scale drawing shows a thin diverging lens with a focal length of -8 cm (focal points labeled $F_D$) placed 20 cm in front of a converging lens with a focal length of 5 cm (focal points labeled $F_C$) is shown in the scale drawing. A 10-cm-tall object shaped like an arrow is placed 15 cm to the left of the diverging lens.

(a) Draw the final image resulting from light from the object that has traveled through both lenses. Show the three principal rays for each step using a ruler.

(b) Estimate the distance between the object and the final image from your drawing. Also estimate the magnification.

(c) Calculate the answers to (b) using the numbers given. Do they agree with your estimates?
8. In the circuit pictured, the switch is closed at time $t = 0$.

(a) Sketch a qualitatively correct graph of the current as a function of time, putting at least one numerical value on each axis.

(b) How long does it take the current to reach 90% of its maximum value?

9. Consider the situation shown in the figure. An electric field of 300 V/m is confined to a circular area 10 cm in diameter and directed perpendicular to the page toward you. The field is increasing at a rate of 20 V/m·s.

(a) What are the direction and magnitude of the magnetic field at point $P$, 15 cm from the center of the circle?

(b) If the electric field is created by a parallel plate capacitor with circular plates lying parallel to the plane of the page above and below it, what current must be flowing into the capacitor?
10 A light ray enters a prism with a square cross-section and index of refraction \( n = 1.3 \) at a 75° angle to one of the sides, and travels out an adjacent side as shown. Find the angle \( \theta_1 \) in the figure. Show your work clearly.

11 A concave mirror has a radius of curvature of 40 cm. A 10-cm-tall object is placed 30 cm from the center of the mirror. Answer the following, showing your work if there is work to be shown.

(a) The image is ________ cm from the mirror.

(b) The image is \( \underline{in \ front \ of} \quad \underline{behind} \) the mirror. (Circle the correct underlined response.)

(c) The image is \( \underline{real} \quad \underline{virtual} \).

(d) The image height is _______________ cm.

(e) The image is \( \underline{upright} \quad \underline{inverted} \).
A wire coming from far away carries a current $I$ which is charging a metal sphere. Use the Ampere-Maxwell law to calculate $\oint \mathbf{B} \cdot d\mathbf{\ell}$ around the loop $L$, shown in the figure, which is concentric with the sphere and has a radius $R$ that is larger than the sphere’s radius. (Note: the surface area of a sphere is $4\pi r^2$.)