Equipotential surfaces and electric field lines surrounding several configurations of conductors are drawn by placing the conductors inside a partially conducting sheet and measuring current flows between pairs of points on the sheet.

**Theory**

A test charge $q$ placed in the vicinity of a charge distribution will experience a force vector $\vec{F}_q$ that is proportional to $q$. This can be expressed by the relation $\vec{F}_q = q\vec{E}$, which defines the electric field vector at the location of the test charge as $\vec{E} = \vec{F}_q / q$.

**Q1:** What are the SI units of $\vec{E}$? _________________

This suggests a very useful way to think about the direction of $\vec{E}$ at a certain location in space: it is the direction that a positive test charge would experience a force if it were placed in that location.

The upper figure at the right shows a collection of charges producing an electric field $\vec{E}$ at a certain location in space.

The lower figure at the right shows a positive test charge $q$ located at the corresponding point from the figure above. The charge experiences a force $\vec{F} = q\vec{E}$ along the same direction as $\vec{E}$.

We can think of the vector field $\vec{E}$ due to a charge distribution as the collection of arrows, one for each location in space. The figure at the right shows the direction of the electric field due to two positive charges. An accurate depiction of the electric field should also show the arrows getting longer as they get closer to the charges, but that would produce a cluttered picture, so we omit this feature in the figure.

The above way of displaying a vector field is somewhat difficult to draw. A simpler way to visualize the field is by drawing electric field lines. The “lines” are really curves, drawn is such a way that they are tangent to $\vec{E}$ at each point. The direction of the arrowheads of $\vec{E}$ is indicated with an arrowhead on each line, and the density of the lines indicates the field strength. The electric field lines for the same situation are shown at the right.
**Q2:** An electric field is shown. Sketch enough electric field lines (with arrowheads) to fill up the figure. The density of field lines should be proportional to the field strength.

In this lab we would like to draw the electric field lines generated by certain charge distributions. Unfortunately, both $\vec{E}$ and $\vec{F}_q$ are difficult to measure directly in the laboratory, so we won’t sketch field lines the way you did in question Q2. (Imagine trying to move a tiny charge around in space while measuring the force vector at each location.) It is much easier to measure the potential difference $V_B - V_A$ (measured in volts in SI units) between two points in space, $A$ and $B$. This is defined by the integral

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s},$$

where the integral is computed along a path in space connecting $A$ and $B$. The potential difference measures the degree to which the $\vec{E}$ vectors tend to point more in one direction (parallel to $d\vec{s}$) or the other (anti-parallel to $d\vec{s}$) along the chosen path. In fact, because all electric fields are conservative, the particular path taken does not affect the value of the integral, and the end points $A$ and $B$ alone determine the value of the integral.

**Q3:** Consider points $A$, $B$, and $C$ on the diagram in question Q2.

$V_B - V_C$ is positive negative approximately zero (circle the best answer)
$V_B - V_A$ is positive negative approximately zero (circle the best answer)

**Q4:** Plot and label points $X$ and $Y$ on the diagram in question Q2 and connect them with a path so that it is very clear to your instructor that $-\int_X^Y \vec{E} \cdot d\vec{s}$ is positive along your path.
In the lab we will use a voltmeter to measure the potential difference $V_B - V_A$ between a pair of points $A$ and $B$. If point $B$ is 1V above point $A$ in electric potential, the voltmeter will read “1.00 V”. The collection of all points that are the same number of volts above point $A$ are collectively called an **equipotential surface** in space, or an **equipotential curve** in two dimensions.

Now suppose that very close points $C$ and $D$ are displaced from each other by a small vector $\vec{s}$ and satisfy $V_D - V_C = 0$. If $\vec{E}$ varies smoothly throughout space, as it does under realistic conditions, then it will be approximately constant in the region between $C$ and $D$. In this case we have $0 = -\int_C^D \vec{E} \cdot d\vec{s} \approx -\int_C^D \vec{E} \cdot d\vec{s} = -\vec{E} \cdot \vec{s}$, so that $\vec{E}$ and $\vec{s}$ are perpendicular whenever $\vec{E} \neq \vec{0}$ in the region. Thus, equipotential surfaces (along $\vec{s}$) and field lines (along $\vec{E}$) are always perpendicular where they meet.

When we draw a collection of field lines and equipotentials in a figure, we conventionally draw equipotential lines that are equally spaced in potential difference. In other words, if the potential difference between one equipotential line and a neighboring line is 2 V, then the separation between all neighboring equipotential lines should be 2 V. Following this convention, the density of equipotential lines will be proportional to the density of electric field lines at each point in a figure. An easy way to ensure this is to draw in the field lines so that adjacent field lines and equipotential lines bound “curvilinear squares”. The average length of one pair of opposite sides should be approximately equal to the average length of the other pair of opposite sides, forming approximate squares as shown in the figure to the right.

We have described some of the properties of electric field lines and equipotentials. The complete set of rules and conventions for drawing collections of electric field lines and equipotentials are given below.

**Properties of Electric Field Lines and Equipotential Curves and Surfaces**

1. Electric field vectors are tangent to electric field lines.
2. The field lines begin on a positive charge and end on a negative charge. If there is an excess of one type of charge, some lines will begin or end infinitely far away.
3. The field line arrowheads point from positive charges and toward negative charges.
4. The number of field lines that leave a positive charge or that end on a negative charge is proportional to the magnitude of the charge.
5. No two field lines can cross.
6. Electric field lines are perpendicular to equipotential surfaces where they intersect.
7. The closer the electric field lines are to each other, the stronger the electric field is in that region.
Our method for drawing electric field lines will be to first find a collection of points at
the same number of volts above a certain point in space. These points can be connected to
make an equipotential curve. After constructing several equipotential curves, we will
draw a series of curves (the field lines) that meet the equipotential curves at right angles,
while being careful to follow each of the seven properties given above.

**Procedure**

1. Install the electrode board (shown at the right) consisting of two small circles on the bottom of the apparatus, using the knobs to secure it in place. (The brown side faces the board, the grey side faces away from the board.) Place a piece of white paper on the top side of the board and slip the edges under the rubber washers at the corners. Use the plastic template to trace the locations of the electrodes on the paper, making sure that the positions of the actual electrodes and your tracings correspond.

2. Turn the power supply knob to the off position and plug it into an outlet. Connect its red terminal to the terminal on the right side of the apparatus using a red banana plug cable. Connect the power supply’s black terminal to the terminal on the left side of the apparatus using a black banana plug cable. The red terminal on the power source is the positive side, and the black is negative. Mark the electrode locations on your paper with “+” or “-“. The + terminal will now be held at a higher potential than the – terminal when the power supply dial is turned on.

3. Connect the terminal on the plastic probe to the \( \text{V\Omega} \) terminal of the digital multimeter with a red banana plug, and connect the terminal on the left-hand side of the board to the COM terminal with a black banana plug. Turn the multimeter dial to the 20 V selection.

4. Slide the arms of the probe around the board and paper. Turn the power supply on all the way. Move the probe to locate a position that gives a 1.00 V reading. This a position that is 1V higher than the “-“ terminal. Mark the location of this position with your pencil through the hole in the probe tip. Repeat for a variety of other positions, enough so that you can visualize the shape of the 1V equipotential curve. Connect the dots with a smooth curve, and label it “1V”.

5. Repeat for the 2V, 3V, 4V, and 5V equipotential curves, plotting enough points to sketch a sizeable equipotential curve for each. Sketch each curve and label it. Your curves should cover the entire sheet of paper.

6. Repeat steps 1 through 5 for the electrode board with the small circle and the long bar, shown at the right. Use a fresh sheet of paper.
Analysis

1. Place a blank sheet over each of your sheets and trace the terminal shapes and equipotential lines so that each partner has a copy of each sheet.

2. Sketch in enough electric field lines to fill up each of your sheets, adding an arrowhead to each and following properties 1-7 given in the theory section, being careful to form “curvilinear squares”.

3. Sketch the electric field lines for the configuration of three equal magnitude charges shown. Two are positive, and one is negative. Let eight field lines touch each charge.
Type your extremely lucid answers to questions 4-6 on a separate sheet of paper.

4. Why are the equipotential lines near the conductor surfaces parallel to the edge?

5. Explain why it is impossible for two electric field lines to cross. Your explanation should be phrased in terms of a test charge and forces on it.

6. Assume that the potential difference between the two conducting terminals in this experiment is $V_+ - V_- = 7V$.

(a) If $V_+ - V_- = 7V$ and the electrodes are connected by a conductor, in which direction do the electrons flow? (+ to − or − to +?)

(b) In which direction does the current flow?

(c) Is it possible for the “+” terminal to have no charge on it in this experiment? If so, what sign of charge would the negative terminal have to have?

(d) Is it possible for the “+” terminal to have a negative charge on it in this experiment? If so, what has to be true of the charge on the “−” terminal?

(e) If the “+” terminal could have a negative charge on it, but $V_+ - V_- = 7V$, why would the electrons run toward the “+” terminal when they are connected by a conductor? Wouldn’t they want to avoid the negative charge that resides there?