

Danville Phone Company  
200 W. Roush Street  
Danville, KE, 44444

February 27, 2009

Discrete Mathematicians  
600 W. Walnut Street  
Danville, KY 40244

Dear Discrete Mathematicians:

Good morning. I received your names from a friend of mine, Cigol Setag with the Circuit Design Company. I recently spoke with him about a problem I am having with my company, and he told me that your mathematical brilliance could help.

As you might already know, our city of Danville is vastly growing in size, and we will be getting our own area code soon. However, we are trying to determine whether or not one area code will suffice or if we will need to introduce two area codes. Mathematicians have projected that the number of Danville residences and businesses needing phone numbers will reach 1 million within the next five years. If each residence and business in the community has their own phone number, will one area code suffice?

There are obviously some restrictions on phone numbers. Each phone number is 7 digits. Phone numbers beginning with the digit 1 or the digit 0 are prohibited. Also, phone numbers cannot begin with 9-1-1, 3-1-1, 6-1-1, 4-1-1, 8-0-0, 9-0-0, or 5-5-5. How many phone numbers are there that meet these conditions?

If possible, we would also like to avoid phone numbers containing the sequence 9-1-1 anywhere in the number. In other words, the phone number 238-9111 would be prohibited because it contains a 9 as its 4<sup>th</sup> digit and a 1 as its 5<sup>th</sup> and 6<sup>th</sup> digits. If we also consider this restriction, how many phone numbers could we produce with only one area code?

Furthermore, would it be possible to avoid phone numbers that would require a user to dial a 9 before dialing two ones? For example, the phone number 293-1401 requires users to dial a 9 before dialing two ones, even though the digits are not dialed sequentially. If we consider this further restriction, how many phone numbers will exist with only one area code?

I would be very grateful if you would submit the full details on your efforts in a typed response so that, in the future, I will hopefully be able to solve similar problems. I would appreciate an answer as soon as possible, but certainly no later than March 13<sup>th</sup>.

Yours Sincerely,

O.N. Gutnic  
Danville Phone Company