Homework 2: Due Wednesday, October 15, 2008 Math 140
Fall 2008 Dr. Wiglesworth

Name: _____

You must *show all work* to receive full credit.
Written homework is expected to be legible and grammatically correct.
Answers and assertions must be fully explained and justified.
Your solutions must be clear, concise, and easy to follow.

1. Find the intervals for which $f(x) = \begin{cases} x^2 - 2x^2 & \text{if } x \leq 2 \\ -x^2 - 25 & \text{if } x > 2 \end{cases}$ is negative. Express your final answer in interval notation.

2. State all discontinuities in the function below. Then state what type of discontinuity each is, making sure to support your answer using the definitions of each of the discontinuities.

$$f(x) = \begin{cases} \frac{3x}{x+2} & \text{if } x < 1 \\ \frac{x-2}{x^2-4} & \text{if} x \geq 1 \end{cases}$$

3. Let $f(x) = \sqrt{2 - x^2}$, $L = -7$, $c = 5$, and $\epsilon = 0.01$. Find the largest value of δ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \epsilon$.

4. In 1960, H. von Forester suggested that the human population could be measured by

$$P(t) = \frac{179 \times 10^9}{(2027 - t)^{0.99}}.$$

The time t is measured in years where $t = 1$ corresponds to the year A.D. 1. Calculate $\lim_{x \to 2027^-} P(t)$, and explain what this means in real-world terms.

5. Graph $f(x) = x^2 - 1$. Then, using the graph, list the following quantities in orer from least to greatest (make sure you clearly justify your answers by relating them to the graph):

(a) the average rate of change of f on [0,1]

(b) the instantaeous rate of change of f at $x = 1$

(c) the instanteous rate of change of f at $x = 0$

(d) the average rate of change of f on [-1,1]

6. Neatly graph $f(x) = x^2 + 1$ and $g(x) = |x| + 1$ on the same coordinate plane. Analyze the graphs near the point (0,1). What do you observe? Which function is differentiable at this point? Write a short paragraph describing the geometric significance of differentiability at a point.

7. Kentucky Power Company burns coal to generate electricity. The cost C in dollars of removing p percent of the air polutants in the stack emissions is $C(p) = \frac{80{,}000p}{100-p}$.

(a) What is the domain of your function in real world terms? Why?

(b) What is the cost of removing 15 percent of the air pollutants?

(c) What is the cost of removing 50 percent of the air pollutants?

(d) What is the cost of removing 90 percent of the air pollutants?

(e) Find $\lim_{x \to 100^-} C(p)$? What does this mean in real-world terms?