Research Statement

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My research interests are in discrete math and more specifically combinatorics and graph theory. I began research in graph theory as an undergraduate student at Transylvania University. For my senior seminar project, I designed a new course schedule for the Natural Sciences Division using graph colorings. This research project sparked my interest in the subject, and once in graduate school, I continued to focus my research in areas of graph theory. In this statement, I will concentrate on describing areas in which I have worked, results obtained, and how I imagine future work will proceed.

Unit Bar-Visibility Graphs

My current focus, and the focus of my dissertation, is unit bar-visibility graphs. A bar-visibility graph (BVG) is a graph whose vertices can be represented in the plane by disjoint horizontal line segments (or bars), such that two vertices are adjacent if and only if there is an unobstructed, non-degenerate, vertical band of visibility between the corresponding bars. A unit bar-visibility graph (UBVG) is a bar-visibility graph in which all bars have the same unit length. Alice Dean and Natalia Veytsel [2] first introduced UBVGs in 2003. In their paper, Dean and Veytsel characterize the trees that are UBVGs and several other small classes of graphs. In 2004, Dean, Ellen Gethner, and Joan Hutchinson [1] published an extended abstract that provides a characterization of triangulated polygons that are UBVGs. Despite a full characterization of BVGs [7, 10]), no simple characterization is known for UBVGs. In fact, there are many classes of unit bar-visibility graphs that have not been characterized.

Very-large-scale integration (VLSI) design motivated the study of bar-visibility graphs. VLSI is the process of creating integrated circuits by combining many transistor-based circuits into a single chip. If two wires cross, a short can occur. Therefore, the circuits need to lay in such a way that the connections can be appropriately made with no crossing wires. The bars of a BVG represent the circuits, and the bands of visibility between the bars represent the wires. However, as bar lengths greatly differ, BVGs are not as useful to VLSI applications. Restricting the length of the bars by requiring all bars to have the same length proves more useful since many circuits in VLSI have near the same size.

My main results are on UBVGs with reach less than two, where the reach of a UBVG layout is the distance from the left-endpoint of the leftmost bar to the right-endpoint of the rightmost bar. Reach less than two graphs also can be considered 2-stack visibility graphs, a special type of 2-stack graph. A k-stack graph is a graph with a layout consisting of a total order of the vertices, and a partition of the edges into k sets of pairwise non-crossing edges. These graphs were first introduced by Ollmann [6] and Heath *et al* [3, 4]. Wigderson [9] showed that determining whether a graph is a 2-stack graph is NP-complete.

I have obtained a characterization of all unit bar-visibility graphs with reach less than two, and therefore all two-stack visibility graphs. All UBVGs with reach less than two must have a Hamiltonian path and a labeling of the vertices that interacts with this path in a special way. I am currently trying to obtain a polynomial-time algorithm for finding this Hamiltonian path using the restrictions placed on the path by the labeling. However, my main focus is on finding a polynomial-time algorithm for induced subgraphs of grids. The Hamilton path problem for induced subgraphs of grids is known to be NP-complete [5]. Therefore, finding a polynomial-time algorithm on these subgraphs will show the difference between the two problems.

Other future goals on unit bar-visibility graphs are to characterize unit bar-visibility graphs with higher reaches. The characterization found for UBVGs with reach less than two will definitely help in doing this. If a graph is a UBVG with reach less than k, it must have a vertex partition into no more than k-1 groups such that each of the partitions has a Hamiltonian path. These partitions must interact in a special way. Furthermore, I would also like to characterize all near triangulations that are unit bar-visibility graphs.

Game Acquisition Number on Graphs

The game acquisition number of a graph G, denoted $a_g(G)$, is a parameter placed on a graph. The game begins on an undirected weighted graph in which unit weight is placed on all vertices. Players make acquisition moves by shifting all the weight from a vertex v to a neighbor with weight less than or equal to the weight of v. The game is played by two players, a maximizer and a minimizer. The maximizer wants to maximize the number of weighted vertices in the final independent set, while the minimizer wants to minimize the number of weighted vertices. The game-acquisition number is the number of weighted vertices that remain after the maximizer and minimizer alternately make acquisition moves, with the minimizer moving first. Therefore, $a_g(G)$ is the resulting number of weighted vertices, assuming both the maximizer and minimizer play optimally. In other words, the maximizer can guarantee $a_g(G)$ weighted vertices will remain, and the minimizer can guarantee that no more than $a_g(G)$, weighted vertices will remain.

This past summer, I participated in a Research Experience for Graduate Students at the University of Illinois at Urbana-Champaign. I began working on this problem with Kevin Milans, Chris Stocker and Professor Doug West. We obtained results that we are currently revising.

Thickness of Sphere of Influence Graphs

Sphere of Influence graphs were first introduces by Toussaint [8] and have applications to pattern recognition and computer vision. A sphere-of-influence graph is defined to be a set of points, each with an open ball centered about it of radius equal to the distance between that point and its nearest neighbor. Two points in the graph are adjacent if their open balls intersect. The closed sphere of influence graph (CSIG) is defined similarly using closed disks. No upper bound was known for the thickness of SIGS. Professor André E. Kézdy, Adam Jobson, and I were able to show the thickness of a CSIG is strictly less than eighteen. We did this by bounding the arboricity of the graphs using Nash-Williams result.

Other Interests

I have broad interests in graph theory. I am interested in several other types of visibility problems such as bar-k-visibility graphs and the visibility number of graphs. Topological graph theory also interests me, particularly problems on planar graphs and doubly linear graphs. Furthermore, I would like to study more areas of discrete geometry in the near future.

References

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