CHAPTER 7
Work and Energy

7.1 WORK DONE BY A FORCE

7.1 A force of 3 N acts through a distance of 12 m in the direction of the force. Find the work done.
Force and displacement are in the same direction, so \( W = Fx = (3 \text{ N})(12 \text{ m}) = 36 \text{ J} \).

7.2 A horizontal force of 25 N pulls a box along a table. How much work does it do in pulling the box 80 cm?
Work is force times displacement through which the force acts. Here, force is in the same direction as the displacement, so \( W = (25 \text{ N})(0.80 \text{ m}) = 20 \text{ J} \).

7.3 A child pushes a toy box 4.0 m along the floor by means of a force of 6 N directed downward at an angle of 37° to the horizontal. (a) How much work does the child do? (b) Would you expect more or less work to be done for the same displacement if the child pulled upward at the same angle to the horizontal?
(a) \( W = F \cos \theta = 6(4)(0.80) = 19.2 \text{ J} \). (b) Less work; since the normal force on the block is less, the friction force will be less and the needed \( F \) will be smaller.

7.4 Figure 7-1 shows the top view of two horizontal forces pulling a box along the floor: (a) How much work does each force do as the box is displaced 70 cm along the broken line? (b) What is the total work done by the two forces in pulling the box this distance?

\[ \text{Fig. 7-1} \]

(a) In each case take the component of the force in the direction of the displacement:
\( (85 \cos 30° \text{ N})(0.70 \text{ m}) = 51.5 \text{ J} \), \( (60 \cos 45° \text{ N})(0.70 \text{ m}) = 29.7 \text{ J} \). (b) Work is a scalar, so add the work done by each force to give 81.2 J.

7.5 A horizontal force \( F \) pulls a 20-kg carton across the floor at constant speed. If the coefficient of sliding friction between carton and floor is 0.60, how much work does \( F \) do in moving the carton 3.0 m?
Because horizontal speed is constant, the carton is in horizontal equilibrium: \( F = f = \mu N \). Normal force is the weight, \( 20(9.8) = 196 \text{ N} \). Therefore \( W = Fx = 0.60(196)(3.0) = 293 \text{ J} \).

7.6 A box is dragged across a floor by a rope which makes an angle of 60° with the horizontal. The tension in the rope is 100 N while the box is dragged 15 m. How much work is done?
Only the horizontal component of the tension, \( T_x = 100 \cos 60° \), does work. Thus, \( W = T_x x = (100 \cos 60°)(15) = 750 \text{ J} \).

7.7 An object is pulled along the ground by a 75-N force directed 28° above the horizontal. How much work does the force do in pulling the object 8 m?
The work done is equal to the product of the displacement, 8 m, and the component of the force that is parallel to the displacement, \( (75 \text{ N}) \cos 28° \).

\[ \text{work} = [(75 \text{ N}) \cos 28°](8 \text{ m}) = 530 \text{ J} \].

7.8 The coefficient of kinetic friction between a 20-kg box and the floor is 0.40. How much work does a pulling force do on the box in pulling it 8.0 m across the floor at constant speed? The pulling force is directed 37° above the horizontal.
The work done by the force is \( xF \cos 37° \), where \( F \cos 37° = f = \mu N \). In this case \( F_x = mg - F \sin 37° \),

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so that \( F = \mu mg \cos 37^\circ \). For \( \mu = 0.40 \) and \( m = 20 \text{ kg} \), \( F = 75.4 \text{ N} \) and \( W = (75.4 \cos 37^\circ)(8.0) = 482 \text{ J} \).

7.9 Repeat Prob. 7.8 if the force pushes rather than pulls on the box and is directed 37° below horizontal.

\( W = (F \cos 37^\circ)(x) = F \cdot x \); thus \( F \cos 37^\circ = \mu F_N \), as in Prob. 7.8; but now \( F_N = mg + F \sin 37^\circ \); solve for \( F \):

\( F = \mu mg \cos 37^\circ - \mu \sin 37^\circ = 140 \text{ N} \) and \( F = 112 \text{ N} \). Thus \( W = 112(8.0) = 896 \text{ J} \). [This larger value for the work accords with Prob. 7.3(b).]

7.10 How much work is done against gravity in lifting a 3-kg object through a distance of 40 cm?

\( W = (mg)(h) \), so \( W = (3)(9.8)(0.40) = 11.8 \text{ J} \).

7.11 How much work is done against gravity in lifting a 20-lb object through a distance of 4.0 ft?

\( W = (weight)(h) = (20 \text{ lb})(4 \text{ ft}) = 80 \text{ ft} \cdot \text{lb} \).

7.12 A 4-kg object is slowly lifted 1.5 m. (a) How much work is done against gravity? (b) Repeat if the object is lowered instead of lifted.

\( W = (mg)(h) \), so \( W = (4)(9.8)(1.5) = 58.8 \text{ J} \).

7.13 A 400-lb load of bricks is to be lifted to the top of a scaffold 28 ft high. How much work must be done against gravity to lift it?

\( W = (mg)(h) \), so \( W = (400 \text{ lb})(28 \text{ ft}) = 11200 \text{ ft} \cdot \text{lb} \).

7.14 A block moves up a 30° incline under the action of certain forces, three of which are shown in Fig. 7.2. \( F_1 \) is horizontal and of magnitude 40 N. \( F_2 \) is normal to the plane and of magnitude 20 N. \( F_3 \) is parallel to the plane and of magnitude 30 N. Determine the work done by each force as the block (and point of application of each force) moves 80 cm up the incline.

\( W = (F \cdot d) \), so \( W = (40 \text{ N})(0.80 \text{ m}) = 32 \text{ J} \). (Note that the distance must be expressed in meters.)

The component of \( F_2 \) in the direction of the displacement is \( F_2 \cos 30^\circ = (40 \text{ N})(0.866) = 34.6 \text{ N} \). Hence the work done by \( F_2 \) is \( (34.6 \text{ N})(0.80 \text{ m}) = 27.8 \text{ J} \). (Note that the distance must be expressed in meters.)

The component of \( F_3 \) in the direction of the displacement is \( 30 \text{ N} \). Hence the work done by \( F_3 \) is \( (30 \text{ N})(0.80 \text{ m}) = 24 \text{ J} \).

7.15 Compute the useful work done by an engine as it lifts 40 L of tar 20 m. One cubic centimeter of tar has a mass of 1.07 g.

\( W = Mgh \), where \( M \) is the total mass of tar. Since \( M = (40 \text{ L})(10^3 \text{ cm}^3/L)(1.07 \times 10^{-3} \text{ kg/cm}^3) = 42.8 \text{ kg} \), \( W = (42.8 \text{ kg})(9.8 \text{ m/s}^2)(20 \text{ m}) = 8389 \text{ J} = 8.389 \text{ kJ} \).
7.16 A uniform rectangular marble slab is 3.4 m long and 2.0 m wide. It has a mass of 180 kg. If it is originally lying on the flat ground, how much work is needed to stand it on end?

\( W = (mg)h \), where \( h \) is the height through which the center of mass is raised. \( W = (180 \text{ kg})(9.8 \text{ m/s}^2)(1.7 \text{ m}) = 30 \text{ kJ} \).

7.17 In Fig. 7-3, evaluate the work done by the weight \( mg \) acting on a particle of mass \( m \), as the particle is moved (by the application of other forces) from: (a) \( A \) to \( B \); (b) \( B \) to \( A \); (c) \( A \) to \( B \) to \( C \); (d) \( A \) to \( C \) directly; (e) \( A \) to \( B \) to \( C \) to \( A \).

(b) \( W_{BA} = -W_{AB} = mgy \)

(c) \( W_{ABC} = W_{AB} + W_{BC} = -mgy + 0 = -mgy \)

(d) The component of force in the direction of motion is \(-mg \cos \phi\) and \( AC = \Delta s = y/(\cos \phi)\).

\[ W_{AC} = (-mg \cos \phi) \left( \frac{y}{\cos \phi} \right) = -mgy \]

(e) \( W_{ABCA} = W_{AB} + W_{BC} + W_{CA} = -mgy + 0 + mgy = 0 \)

\( F_x (N) \) vs. \( x \) curve (Fig. 7-4).

7.18 The \( x \)-directed force that acts on an object is shown as a function of \( x \) in Fig. 7-4. Find the work done by the force in the interval: (a) \( 0 \leq x \leq 1 \text{ m} \), (b) \( 1 \leq x \leq 3 \text{ m} \), (c) \( 0 \leq x \leq 4 \text{ m} \).

\( W = 2.5 \text{ J} \).

7.19 The \( x \)-directed force that acts on an object is shown as a function of \( x \) in Fig. 7-5. Find the work done by the force in the interval: (a) \( 0 \leq x \leq 3 \text{ cm} \), (b) \( 3 \leq x \leq 5 \text{ cm} \), (c) \( 0 \leq x \leq 6 \text{ cm} \).

\( W = (0.03 \text{ m})(5 \text{ N})/2 = 0.075 \text{ J} \); (b) \( W = -0.02(3)/2 = -0.03 \text{ J} \); (c) The work between 5 and 6 cm is \( 0.01(3) = 0.03 \text{ J} \); adding this to (a) and (b) gives a total work in the 6-cm interval of \( 0.075 + 0.03 = 0.105 \text{ J} \).

7.20 If the \( x \)-directed force exerted on a cart by a boy varies with position as shown in Fig. 7-6, how much work does the boy do on the cart?

The total work is the area under the curve. By counting entire squares and parts of squares we estimate about 34 squares under the curve, so work is about \( 34 \text{ squares} \times (40 \text{ J/square}) = 1360 \text{ J} \).
7.21 How much work is done by a force of 40 N acting 37° above the horizontal in pulling a block 8 m along a horizontal surface?

\[ W = F \cdot s = Fs \cos \theta = (40 \text{ N})(8 \text{ m})(0.8) = 256 \text{ J}. \]

7.22 In order to lift a 5.0-kg child through a vertical distance of 40 cm, how much work must be done?

\[ W = F \cdot s = (5 \times 9.8)(0.40) = 19.6 \text{ J}. \]

7.23 If \( A = A_x i + A_y j + A_z k \) and \( B = B_x i + B_y j + B_z k \), and \( C = C_x i + C_y j + C_z k \), find \((a) A \cdot B \), and \((b) B \cdot C \).

\[ (a) A \cdot B = A_x B_x + A_y B_y + A_z B_z \]

By Prob. 1.85, a dot product of two vectors can be evaluated by componentwise multiplication. Thus:

\[ (a) A \cdot B = A_x B_x + (0)(B_y) + (0)(B_z) = A_x B_x \]

\[ (b) B \cdot C = B_x C_x + B_y C_y + B_z C_z \]

7.24 Find \( A \cdot B \) if \( A = 3i - 4j \) and \( B = 6j + 2k \).

\[ A \cdot B = (3)(0) + (-4)(6) + (0)(2) = -24 \]

If \( A \) is a force in newtons and \( B \) is a displacement in meters, \( A \cdot B \) would represent work done in joules.

7.25 Compute \( C \cdot D \) if \( C = 3j - 2k \) and \( D = -8i + 5k \).

\[ C \cdot D = (0)(-8) + (3)(0) + (-2)(5) = -10 \]

7.26 A constant resultant force \( F = F_x i + F_y j + F_z k \) acts on an object to give it a displacement from the origin \( s = x_i + y_j + z_k \). Give two equivalent expressions for the work done on the object.

\[ W = F \cdot s; \text{ and, according to Prob. 1.83, we may write} \]

\[ W = F \cdot s = F \cos \theta, \text{ where} \]

\[ F = \sqrt{F_x^2 + F_y^2 + F_z^2}, \]

\[ s = \sqrt{x_i^2 + y_j^2 + z_k^2}, \text{ and} \theta \]

is the smaller angle between \( F \) and \( s \); or

\[ F \cdot s = F_x x_i + F_y y_j + F_z z_k \]

7.27 A coin of mass \( m \) slides a distance \( D \) along a tabletop. If the coefficient of friction between the coin and table is \( \mu \), find the work done on the coin by friction.

\[ \text{From the definition,} \quad \Delta W = F \cdot \Delta x. \text{ Because} \quad f \quad \text{opposes the motion, it is directed opposite to} \quad \Delta x. \text{ The magnitude of} \quad f \quad \text{is} \quad \mu mg, \text{ and thus} \quad W = -\mu mg D. \]

7.28 A 5.0-kg box is pulled across the floor at a constant speed of 20 cm/s by a horizontal force. If \( \mu = 0.30 \) between the box and floor, in each second how much work is done by (a) the pulling force? (b) the frictional force? (c) What is the total work per second done on the box?

\[ \text{The speed is constant, thus Newton's first law applies and} \quad F = -f, \text{ where} \]

\[ f = \mu mg = 0.30(5.0)(9.8) = 14.7 N. \]

\[ \Delta W = F \cdot \Delta x \quad \text{yields} \quad W = (14.7 N)(0.20 \text{ m}) = 2.94 J. \text{ (The displacement is the distance moved in 1 s, } 0.20 \text{ m.}) \]

\[ \Delta W = f \cdot \Delta x = -F \cdot \Delta x, \text{ which gives} \]

\[ W = -2.94 J. \]

(c) The total work is the sum of parts (a) and (b), in this case zero.

7.2 WORK, KINETIC ENERGY, AND POTENTIAL ENERGY

7.29 What is the work-energy principle?

\[ \text{The work-energy principle for a particle states that the work} \quad W \quad \text{done by the resultant force acting on the particle is equal to the change in the particle's kinetic energy:} \]

\[ W_{i-f} = K_f - K_i = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2. \]

(Sometimes we write KE instead of \( K \).)

7.30 Prove the work-energy principle for a particle moving at constant acceleration (under a constant force) along a straight line.

\[ v_f^2 = v_i^2 + 2as \]

\[ \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + Fs \]

\[ K_f = K_i + W_{AB} \quad \text{or} \quad W_{AB} = K_f - K_i \]

7.31 How is the work-energy principle expanded to include the potential energy associated with conservative forces (such as gravity)?
Let \(W_p\) represent the work done by those forces which are treated in terms of potential energy, \(U\) (or \(PE\)). Let \(W\) represent the work due to all other forces. From the work-energy principle, \(W_{f}\) = \(W_{p}\) = \(\Delta K\). From the definition of \(U\): \(-W_{f,\nu}\) = \(U_{f} - U_{i} = \Delta U\). Then \(W_{f,\nu} = (U_{f} - U_{i}) + (K_{f} - K_{i}) = \Delta U + \Delta K\) is the modified form of the work-energy principle when potential energy is included.

### 7.32

A car is moving at 100 km/h. If the mass of the car is 950 kg, what is its kinetic energy?

\[
\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2} (950 \text{ kg}) \left( \frac{10^3 \text{ m}}{3600 \text{ s}} \right)^2 = 3.67 \times 10^4 \text{ J} = 0.367 \text{ MJ}
\]

### 7.33

Outstanding performances for a number of athletic events are listed below. Neglecting air resistance and assuming that each projectile is launched at the optimum 45° elevation angle, calculate the initial kinetic energy for each case.

- (a) Shot put: mass = 7.26 kg, distance thrown = 22.0 m.
- (b) Discus throw: 2.00 kg, 70.9 m.
- (c) Hammer throw: 7.26 kg, 79.3 m.
- (d) Javelin throw: 0.800 kg, 94.6 m.
- (e) Long jump: 60.0 kg, 8.90 m.
- (f) Baseball throw: 0.145 kg, 130 m.

We recall that the maximum range for a projectile launched with speed \(v_0\) from ground level is given by \(R_{\text{max}} = \frac{1}{2}g\left(\frac{2v_0^2}{g}\right)\). We use this expression, since no information is given on the launch elevations. The initial kinetic energy \(K_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}mgR_{\text{max}}\).

- (a) \(K_0 = 783\) J, (b) \(K_0 = 695\) J, (c) \(K_0 = 2.82\) kJ, (d) \(K_0 = 371\) J, (e) \(K_0 = 2.62\) kJ, (f) \(K_0 = 92.4\) J.

### 7.34

A 150-g mass has a velocity \(v = (2i + 6j)\) m/s at a certain instant. What is its kinetic energy?

\[
K = \frac{1}{2}mv^2 = \frac{1}{2}(0.150)(2^2 + 6^2) = 3.0\text{ J}
\]

### 7.35

The velocity of an 800-g object changes from \(v_i = 3i - 4j\) to \(v_f = (-6i + 2j)\) m/s. What is its change in kinetic energy?

\[
\text{Using } v^2 = v_x^2 + v_y^2 = 40 \text{ and } v_y^2 = 25. \text{ Therefore, change in } K = 0.800(40 - 25)/2 = 6.0\text{ J.}
\]

### 7.36

How large a force is required to accelerate a 1300-kg car from rest to a speed of 20 m/s in a distance of 80 m?

\[
W = \Delta K = \frac{1}{2}mv^2 = \frac{1}{2}(1300 \text{ kg})(20 \text{ m/s})^2 = 260 \text{ kJ}. \text{ But } W = F \cdot s = F(80 \text{ m}); F = 3.25 \text{ kN}.
\]

### 7.37

A crate of mass 50 kg slides down a 30° incline. The crate’s acceleration is 2.0 m/s², and the incline is 10 m long. (a) What is the kinetic energy of the crate as it reaches the bottom of the incline? (b) How much work is spent in overcoming friction? (c) What is the magnitude of the frictional force that acts on the crate as it slides down the incline?

- (a) We denote the crate’s mass and acceleration by \(m\) and \(a\), respectively. After sliding a distance \(s\) from rest, the crate has a kinetic energy \(K\) given by \(K = \frac{1}{2}mv^2 = \frac{1}{2}m(2as) = mas\). With \(m = 50 \text{ kg}, a = 2.0 \text{ m/s}^2, \text{ and } s = 10\text{ m}, \text{ we obtain } K = 1000\text{ J}.

- (b) The work \(W_c\) done on the crate by gravity is given by \(W_c = mgh\), where \(h\) is the vertical distance descended by the crate. For an incline of angle \(\theta\), \(h = s \sin \theta\), so we find \(W_c = mgs \sin \theta\). With \(\theta = 30^\circ\) and the other given values, we find \(W_c = 2450\text{ J}\). The only other force which does work on the crate is friction. If we let \(W_f\) denote the work done by friction, we have \(W_c + W_f = K\). Therefore \(W_f = K - W_c = 1000 - 2450 = -1450\text{ J}\). The work spent in overcoming friction is \(|-1450\text{ J}| = 1450\text{ J}\).

- (c) The work \(W_f\) done by friction is given by \(W_f = -Fs\), so that \(F = -W_f/s = -(1450)/(10) = 145\text{ N}\).

### 7.38

Refer to Prob. 7.37. (a) What is the coefficient of kinetic friction between the crate and the incline? (b) At the base of the incline there is a horizontal surface with the same coefficient of kinetic friction. How far will the crate slide before coming to rest?

- (a) Since the crate remains in contact with the incline, the normal force \(N = mg \cos \Theta\). Then the frictional force \(F_f = \mu_sN = \mu_smg \cos \Theta\). Solving for \(\mu_s\), we obtain \(\mu_s = F_f/(mg \cos \Theta) = 145/[(30)(9.8)(0.866)] = 0.342\).

- (b) On a horizontal surface, the frictional force will be \(F_f = \mu-navbar{s}\). The crate will slide a distance \(s'\) such that the work \(W_f\) done by friction equals the negative of the kinetic energy \(K\). That is, \(-\mu_sms' = -K\), so that \(s' = K/(\mu_sms) = 1000/[(0.342)(30)(9.8)] = 5.97\text{ m}\).

### 7.39

A 1200-kg car going 30 m/s applies its brakes and skids to rest. If the friction force between the sliding tires and the pavement is 6000 N, how far does the car skid before coming to rest?

\[
W = \Delta K = 0 - \frac{1}{2}mv^2 = -\frac{1}{2}(1200 \text{ kg})(30 \text{ m/s})^2 = -540 \text{ kJ}. \text{ But } W = -fx = -(6 \text{ kN})x; x = 90 \text{ m}.
\]
7.40 How much work is done in moving a body of mass 1.0 kg from an elevation of 2 m to an elevation of 20 m, 
(a) by the gravitational field of the earth? (b) by the external agent lifting the body?

\[ W = -\Delta U = -(1.0)(9.8)(20) - (1.0)(9.8)(2) = -176.4 \text{ J} \]

The work is negative because the force opposes the motion.

(b) By Prob. 7.31, \( W' = \Delta K + \Delta U = \Delta K + 176.4 \text{ J} \). Unlike the gravitational work, the external work depends on the change in speed of the body. If the body is unaccelerated (\( \Delta K = 0 \)), then \( W' = 176.4 \text{ J} \), the negative of the gravitational work.

7.41 A 200-kg cart is pushed slowly up an incline. How much work does the pushing force do in moving the object up along the incline to a platform 1.5 m above the starting point if friction is negligible?

Work done by all forces other than gravity equals the combined change in gravitational potential energy and kinetic energy. Since the force \( F \) pushing the cart up the incline is the only such force doing work, we have

\[ W_F = \Delta U + \Delta K = (mgh - 0) + (0) = (200 \text{ kg})(9.8 \text{ m/s}^2)(1.5 \text{ m}) = 2.94 \text{ kJ} \]

7.42 Repeat Prob. 7.41 if the distance along the incline to the platform is 7 m and a friction force of 150 N opposes the motion.

Now we must consider the work done by the frictional force, \( f = 150 \text{ N} \), as well as that done by \( F \) (see Prob. 7.41). Thus we have \( W_F + W_f = \Delta U + \Delta K = 2.94 \text{ kJ} \). Noting that \( W_f = -(150 \text{ N})(7 \text{ m}) = -1050 \text{ J} \), we get \( W_F = 3.99 \text{ kJ} \).

7.43 A ladder that is 3.0 m long and weighs 200 N has its center of gravity 120 cm from the foot. At its top end is a 50 N weight. Compute the work required to raise the ladder from a horizontal position on the ground to a vertical position.

The work done (against gravity) consists of two parts—that needed to raise the center of gravity 1.2 m and that needed to raise the weight at the end through 3.0 m. Therefore, \( W = (200 \text{ N})(1.2 \text{ m}) + (50 \text{ N})(3.0 \text{ m}) = 390 \text{ J} \).

Another method. The center of gravity of the system is at a distance

\[ \bar{x} = \frac{(200 \text{ N})(1.2 \text{ m}) + (50 \text{ N})(3.0 \text{ m})}{250 \text{ N}} = \frac{390}{250} \text{ m} \]

from the foot. To lift 250 N through 390/250 m requires 390 N \( \cdot \) m = 390 J of work.

7.44 A 100 000-lb freight car is drawn 2500 ft up along a 1.2 percent grade at constant speed. Find the work done against gravity.

Work done against gravity is only the increase in gravitational potential energy, \( \Delta U = mgh \), where

\[ h = 0.012(2500 \text{ ft}) = 30 \text{ ft} \]

Thus

\[ \Delta U = (100 000 \text{ lb})(30 \text{ ft}) = 3.0 \times 10^6 \text{ ft} \cdot \text{lb} \]

7.45 If a constant frictional force of 440 lb retards the motion in Prob. 7.44, how much work is done by the pulling force?

Letting \( W_p \) be the work done by the drawbar and \( W_f \) the work done by friction, we have \( W_p + W_f = \Delta U + \Delta K \) (see Prob. 7.31). Since the speed is constant, \( \Delta K = 0 \), and \( W_f = -(440 \text{ lb})(2500 \text{ ft}) = -1.1 \times 10^6 \text{ lb} \cdot \text{ft} \), then \( W_p = \Delta U - W_f = 4.1 \times 10^6 \text{ ft} \cdot \text{lb} \).

7.46 A cylindrical tank 10 m high has an internal diameter of 4 m. How much work would be required to fill the tank with water if the water were pumped in (a) at the bottom and (b) over the top edge?

It takes 1.23 MN of water to fill the tank, since \( pgh^2r^2 = (10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(2 \text{ m})^2(10 \text{ m}) = 1.23 \text{ MN} \). (a) In effect, the center of gravity of a 1.23-MN body of water is raised from height 0 to height 5 m; work required = \( (1.23 \text{ MN})(5 \text{ m}) = 6.15 \text{ MJ} \). (b) The CG must be raised from height 0 to height 10 m (after which it falls by itself to height 5 m): work required = \( 2 \times 6.15 = 12.30 \text{ MJ} \).

7.47 A box weighing 200 N is dragged up an incline 10 m long and 3 m high. The average force (parallel to the plane) is 120 N. (a) How much work is done? (b) What is the change in the potential energy of the box? in its kinetic energy? (c) What is the frictional force on the box?
(a) The work done by the dragging force is \( W_{\text{drag}} = \vec{F} \cdot \vec{s} = 120 \times 10 = 1200 \text{ J} \). (b) The change in potential energy is \( \Delta U = U_f - U_i = wh - 0 = 200 \times 3 = 600 \text{ J} \). (c) Because the box starts and finishes at rest, \( \Delta K = 0 \). The total work done by nonconservative forces is \( W_{\text{total}} = W + W_f \), where \( W \) is the work done on the box by friction. Then

\[
\Delta K + \Delta U = W_{\text{total}} + W \\
0 + 600 = 1200 + W \\
W = -600 \text{ J}
\]

But \( W = -fs \) (the friction force \( f \) opposes the motion of the box), and so

\[
f = \frac{-600}{-10} = 60 \text{ N}
\]

A rock weighing 20 N falls from a height of 16 m and sinks 0.6 m into the ground. From energy considerations, find the average force \( f \) between the rock and the ground as the rock sinks. See Fig. 7-7.

Between \( A \) and \( C \), nonconservative work \( W' = -fh' \) is done on the rock.

\[
\Delta K + \Delta U = W' = 0 + [mg(-h') - mgh] = -fh' \quad f = \frac{20(16.6)}{0.6} = 553 \text{ N}
\]

A pistol fires a 3-g bullet with a speed of 400 m/s. The pistol barrel is 13 cm long. (a) How much energy is given to the bullet? (b) What average force acted on the bullet while it was moving down the barrel? (c) Was this force equal in magnitude to the force of the expanding gases on the bullet?

(a) The kinetic energy of the bullet on leaving the barrel is \( K_f = \frac{1}{2}mv^2 = \frac{1}{2}(0.003)(400)^2 = 240 \text{ J} \).

(b) The work done on the bullet is equal to the change in its kinetic energy. \( W = \vec{F} \cdot x = K_f - K_i \), where \( \vec{F} \) is the average (with respect to distance \( x \)) force exerted on the bullet. Thus,

\[
F = \frac{K_f - K_i}{x} = \frac{240 - 0}{0.13} = 1846 \text{ N}
\]

since the bullet was at rest initially.

(c) No, since there are frictional forces in play as the bullet moves down the barrel.

A bullet having a speed of 153 m/s crashes through a plank of wood. After passing through the plank, its speed is 130 m/s. Another bullet, of the same mass and size but traveling at 92 m/s, is fired at the plank. What will be this second bullet's speed after tunneling through? Assume that the resistance of the plank is independent of the speed of the bullet.

The plank does the same amount of work on the two bullets, and therefore decreases their kinetic energies equally.

\[
\frac{1}{2}m(153)^2 - \frac{1}{2}m(130)^2 = \frac{1}{2}m(92)^2 - \frac{1}{2}mu^2 \\
u^2 = 1955 \\
u = 44.2 \text{ m/s}
\]

A delivery boy wishes to launch a 2.0-kg package up an inclined plane with sufficient speed to reach the top of the incline. The plane is 3.0 m long and is inclined at 20°. The coefficient of kinetic friction between the package and the plane is 0.40. What minimum initial kinetic energy must the boy supply to the package?
The incline is shown in Fig. 7-8. If the package travels the entire length \( s \) of the incline, the frictional force will perform work \(-\mu_k N_s \) on it. Furthermore, the gravitational potential energy of the package will increase by \( mgh \). To reach the top, the package must have initial kinetic energy \( \frac{1}{2}mv^2 \approx mg h + \mu_k N_s \) (see Prob. 7.31). Since \( N = mg \cos \theta \) and \( h = s \sin \theta \), we have \( \frac{1}{2}mv^2 \approx mgs \sin \theta + \mu_k mg \cos \theta \). With \( \theta = 20^\circ \), \( m = 2.0 \text{ kg}, s = 3.0 \text{ m}, \) and \( \mu_k = 0.40 \), we find \( \frac{1}{2}mv^2_{\text{initial}} = (2.0)(9.8)(3.0)[(0.342) + (0.40)(0.940)] = 42 \text{ J} \).

7.52 A driver of a 1200-kg car notices that the car slows from 20 m/s to 15 m/s as it coasts a distance of 130 m along level ground. How large a force opposes the motion?

\[ W_s = -F_s = \Delta K, \] where \( F \) is the force in question and \( s = 130 \text{ m} \). \( \Delta K = \left( \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \right) = \frac{1}{2}(1200 \text{ kg})(225 \text{ m}^2/\text{s}^2 - 400 \text{ m}^2/\text{s}^2) = -105 \text{ kJ} \). Thus \( F_s = 105 \text{ kJ} \) and \( F = 0.81 \text{ kN} \).

7.53 A 10-lb weight slides from rest down a rough inclined plane, 100 ft long and inclined 30° to the horizontal, and gains a speed of 52 ft/s. Find the work done against friction.

\[ W' = \Delta U + \Delta K, \] where in this case \( W' \), the work done by all forces other than gravity, is simply \( W' \), the work done by friction. Then \( W' = (0 - mgL \sin 30^\circ) + \left( \frac{1}{2}mv^2 - 0 \right) \), with \( mg = 10 \text{ lb} \), \( L = 100 \text{ ft} \), \( v = 52 \text{ ft/s} \). Substituting in these values and noting that \( m = 0.311 \text{ slug} \), we obtain \( W' = -80 \text{ ft}-\text{lb} \).

7.54 Redo Prob. 7.53 from an energy balance point of view.

\[ W' = \Delta U + \Delta K, \] where in this case \( W' \), the work done by all forces other than gravity, is simply \( W' \), the work done by friction. Then \( W' = (0 - mgL \sin 30^\circ) + \left( \frac{1}{2}mv^2 - 0 \right) \), with \( mg = 10 \text{ lb} \), \( L = 100 \text{ ft} \), \( v = 52 \text{ ft/s} \). Substituting in these values and noting that \( m = 0.311 \text{ slug} \), we obtain \( W' = -80 \text{ ft}-\text{lb} \).

7.55 A 2000-kg elevator rises from rest in the basement to the fourth floor, a distance of 25 m. As it passes the fourth floor, its speed is 3.0 m/s. There is a constant frictional force of 500 N. Calculate the work done by the lifting mechanism.

\[ W' = \Delta U + \Delta K, \] where \( W' = W_f + W_p \), the work done by the lift force and frictional force, respectively. Then, \( W_f - fh = (Mgh - 0) + \left( \frac{1}{2}Mu^2 - 0 \right) \), where \( M = 2000 \text{ kg}, h = 25 \text{ m}, \) and \( f = 500 \text{ N} \). Substituting in, we get \( W_f = 12.5 \text{ kJ} = 490 \text{ kJ} + 9 \text{ kJ}, \) or \( W_p = 511.5 \text{ kJ} \).

7.56 Redo Prob. 7.56 from an energy-balance point of view.

\[ W_f = -\mu_k mgL = (0 - \frac{1}{2}mv^2), \] or \( v^2 = (25 \text{ m/s})^2 - 2 \mu_k gL = 2(0.8)(9.8 \text{ m/s}^2) \). Solving, \( L = 40 \text{ m} \).

7.57 The coefficient of sliding friction between a 900-kg car and pavement is 0.80. If the car is moving at 25 m/s along level pavement when it begins to skid to a stop, how far will it go before stopping?

\[ W_f = \Delta U + \Delta K, \] where \( W_f \) is the work done by friction, and \( \Delta U = 0 \). Then \( -\mu_k mgL = (0 - \frac{1}{2}mv^2), \) or \( v^2 = (25 \text{ m/s})^2 - 2 \mu_k gL = 2(0.8)(9.8 \text{ m/s}^2) \). Solving, \( L = 40 \text{ m} \).

7.58 In the setup of Fig. 7-9, the coefficient of friction between the mass and the plane is 0.2. (a) How much work is done by the force \( F \) in moving the mass up the plane a distance of 4 m? (b) If the force \( F \) is removed and the mass slides back down, what will its speed be when it reaches its original position?
(a) The weight \( w = 50 \times 9.8 = 490 \text{ N} \) and

\[
\begin{align*}
   w \sin 36.9^\circ &= 294 \text{ N} \\
   w \cos 36.9^\circ &= 392 \text{ N}
\end{align*}
\]

The friction \( f = \mu w \cos 36.9^\circ = 0.2 \times 392 = 78.4 \text{ N} \). Then the work \( W_f = (294 + 78.4) \times 4 = 1.49 \text{ kJ} \), to just reach the top with no residual kinetic energy.

(b) \( \text{PE at top} = 294 \times 4 \times m = 1176 \text{ J} \) \( \text{work against friction} = 78.4 \times 4 = 313.6 \text{ J} \)

\[
\begin{align*}
   (1176 - 313.6) J &= 862.4 \text{ J} = \frac{1}{2}mv^2 \\
   \frac{1}{2}(50)v^2 &= 862.4 \\
   v^2 &= 34.496 \text{ m}^2/\text{s}^2 \\
   v &= 5.87 \text{ m/s}
\end{align*}
\]

7.59 A ski jumper glides down a 30° slope for 80 ft before taking off from a negligibly short horizontal ramp. If takeoff speed is 45 ft/s, what is the coefficient of kinetic friction on the slide?

\[
\begin{align*}
   \text{PE} &= mh = m \times 32 \times 80 \text{ sin 30°} = 1280m \text{ ft} \cdot \text{lb} \\
   \text{KE at takeoff} &= \frac{1}{2}m(45)^2 = 1012.5m \text{ ft} \cdot \text{lb}
\end{align*}
\]

Thus

\[
\begin{align*}
\text{energy dissipated} &= \frac{1280m}{2} \cdot 1012.5m = 675m \text{ ft} \cdot \text{lb} \\
   f \times 80 &= 267.5m \\
   f &= \mu mg \cos 30°
\end{align*}
\]

\[
\mu_s(m \times 32 \times 0.866) \times 80 = 267.5m \\
\mu_s &= \frac{267.5m}{80 \times 32 \times 0.866} = 0.121
\]

7.60 A pump lifts water from a lake to a large tank 20 m above the lake. How much work does the pump do as it transfers 5 m³ of water to the tank? One cubic meter of water has a mass of 1000 kg.

\( \text{Work done by the pump, } W, \text{ equals increase in potential energy of the water. } W = \Delta U = Mgh = (5000 \text{ kg})(9.8 \text{ m/s}^2)(20 \text{ m}) = 9.8 \text{ M}J \). (Compare Prob. 7.46; here the height of the tank is neglected in comparison with 20 m.)

7.3 CONSERVATION OF MECHANICAL ENERGY

7.61 What is the law of conservation of mechanical energy?

\( \text{From Prob. 7.31 we have } W' = \Delta U + \Delta K, \text{ where } W' \text{ is the work done by all nonconservative forces and } U \text{ is the potential-energy function for all the conservative forces. If } W' = 0, \text{ we have } \Delta U + \Delta K = (U_f - U_i) + (K_f - K_i) = 0 \text{ or } U_f + K_f = U_i + K_i \text{ (law of conservation of mechanical energy)} \)

Note that nonconservative forces may be acting (e.g., the normal force exerted by a smooth, curved wire on a bead sliding along it), but so long as they do no work, the conservation law holds.

7.62 Just before striking the ground, a 2.0-kg mass has 400 J of KE. If friction can be ignored, from what height was it dropped?

\[
\begin{align*}
   U_f + K_f &= U_i + K_i \\
   0 + K_f &= mgh + 0 \\
   h &= \frac{K_f}{mg} = \frac{400}{(2.0)(9.8)} = 20.4 \text{ m}
\end{align*}
\]

7.63 A 40-g body starting from rest falls through a vertical distance of 25 cm to the ground. (a) What is the kinetic energy of the body just before it hits the ground? (b) What is the velocity of the body just before it hits the ground?

\( \text{(a) As the body falls its gravitational potential energy is converted to kinetic energy.} \)

\[
\text{PE} = mgh = (0.040 \text{ kg})(9.8 \text{ m/s}^2)(0.25 \text{ m}) = 0.098 \text{ J}
\]

Therefore, the KE just before the body hits the ground is 0.098 J.

\( \text{(b) } KE = \frac{1}{2}mv^2 = 0.098 = \frac{1}{2}(0.04) v^2 \text{ or } v^2 = 4.9 \text{ m/s} \)

7.64 A boy throws a 0.15-kg stone from the top of a 20-m cliff with a speed of 15 m/s. Find its kinetic energy and speed when it lands in a river below.

\[
\begin{align*}
   \text{(PE + KE) at top} &= \text{KE at river} \\
   (0.15 \times 9.8 \times 20) + \frac{1}{2}(0.15)(15)^2 &= \frac{1}{2}(0.15)v^2 \\
   \text{KE at river} &= 29.4 + 16.9 = 46.3 \text{ J} \\
   \frac{1}{2}(0.15)v^2 &= 46.3 \\
   v &= 24.8 \text{ m/s}
\end{align*}
\]

7.65 A 0.5-kg ball falls past a window that is 1.50 m in vertical extent. How much did the KE of the ball increase as it fell past the window? If its speed was 3.0 m/s at the top of the window, what was it at the bottom?
CHAPTER 7

7.65 At sea level a nitrogen molecule in the air has an average translational KE of $6.2 \times 10^{-21}$ J. Its mass is $4.7 \times 10^{-26}$ kg. If the molecule could shoot straight up without striking other air molecules, how high would it rise? What is the molecule’s initial speed?

We first assume that the height is not so great that we have to use the exact law of gravity, $F = GMm/l^2$. Instead we assume that $F = mg$ and $U = mgh$ are valid. If we are wrong, it will be evident from the value of $h$. If the molecule shoots up unopposed, we have $\Delta U + \Delta K = 0$ and the loss of kinetic energy in reaching the highest point must equal the gain in potential energy. Thus $6.2 \times 10^{-21} J = mgh = (4.7 \times 10^{-26} \text{kg})(9.8 \text{ m/s}^2)h$, and $h = 13.5 \text{ km}$. This is small compared with the radius of the earth (6400 km) and our assumption is reasonable. The initial speed of the molecule is obtained from $\frac{1}{2}mv^2 = 6.2 \times 10^{-21} \text{ J}$. $v = 514 \text{ m/s}$.

7.67 If the simple pendulum shown in Fig. 7-10 is released from point $A$, what will be the speed of the ball as it passes through point $C$?

The tension in the cord does no work (refer to Prob. 7.61). When the ball reaches point $C$ it has lost $mgh_a$ in potential energy, where $h_a = 0.75 \text{ m}$, and in its place gained $\frac{1}{2}mv_C^2$ in kinetic energy, with $mgh_a = \frac{1}{2}mv_C^2$. The $m$'s cancel, leading to $v_C^2 = 2gh_a = 2(9.8 \text{ m/s}^2)(0.75) = 14.7 \text{ m}^2/\text{s}^2$, or $v_C = 3.83 \text{ m/s}$.

7.68 Refer to Fig. 7-10. What is the speed of the ball at point $B$?

With point $C$ as the zero reference for potential energy, the conservation law gives $mgh_a + \frac{1}{2}mv_B^2 = mgh_a + \frac{1}{2}mv_C^2$, with $v_a = 0$, $h_a = 0.75 \text{ m}$, and $h_a = (0.75 \text{ m})(1 - \cos 37^\circ) = 0.15 \text{ m}$. Our equation becomes $v_B^2 = 2gh_a - 2gh_b = 11.76 \text{ m}^2/\text{s}^2$, and $v_B = 3.43 \text{ m/s}$.

7.69 A pendulum bob has a mass of $0.5 \text{ kg}$. It is suspended by a cord $2 \text{ m}$ long which is pulled back through an angle of $36.9^\circ$ (Fig. 7-11) and released. Find (a) its maximum potential energy relative to its lowest position, (b) its potential energy when the cord makes an angle of $10^\circ$ with the vertical, (c) its maximum speed, and (d) its speed when the cord makes an angle of $10^\circ$ with the vertical.

(a) $PE_{max} = (0.5 \times 9.8)0.4 = 1.96 \text{ J}$. (b) When $\theta = 10^\circ$,

$h_{10} = 2 - 2 \cos 10^\circ = 2(1 - 0.9848) = 0.0304 \text{ m}$

$PE_{10} = (0.5 \times 9.8) \times 0.0304 = 0.149 \text{ J}$

(c) At the bottom $\frac{1}{2}m(v_{max})^2 = 1.96$ $v_{max} = 2 \times 1.4 = 2.8 \text{ m/s}$

(d) $(1.96 - 0.149)J = \frac{1}{2}m(v_{10})^2$ $v_{10} = 7.24 \text{ m/s}$

7.70 A 0.8-kg pendulum bob on a 2-m cord is pulled sideways until the cord makes an angle of $36.9^\circ$ with the vertical (see Fig. 7-11). Find the work done on the bob and the speed of the bob as it passes through the equilibrium position after being released at rest.

(a) $h = 2 - 1.6 = 0.4 \text{ m}$ work done by gravity = loss in $PE$ loss in $PE = mgh = 0.8 \times 9.8 \times 0.4 = 3.136 \text{ J}$

From the work-energy theorem $\frac{1}{2}mv^2 = mgh$ $v^2 = 2gh = 2 \times 9.8 \times 0.4 = 7.84 \text{ m}^2/\text{s}^2$ $v = 2.8 \text{ m/s}$.

7.71 A toy car starts from rest at position 1 shown in Fig. 7-12(a) and rolls without friction along the loop 12324. Find the smallest height $h$ at which the car can start without falling off the track.

When $h$ has its critical value, the car will just lose contact with the track at position 3. Then, at 3, the
normal force due to the loop vanishes and \( mgh = \frac{1}{2} mv_3^2 \), or \( v_3^2 = gr \). By conservation of energy, the potential energy of the car at position 1 is the same as the kinetic energy of the car at position 3:

\[
mgh = \frac{1}{2} mv_3^2 \quad h = \frac{v_3^2}{2g} = \frac{r}{2}
\]

7.72 Refer to Prob. 7.71. What speed does the car have at position 4?

\( \square \) Applying conservation of energy between positions 1 and 4,

\[
0 + mgh = \frac{1}{2} mv_4^2 + mg(-2r) \quad v_4^2 = 2g(h + 2r) = 5gr \quad v_4 = \sqrt{5gr}
\]

7.73 In the track shown in Fig. 7.13, section AB is a quadrant of a circle of 1.0-m radius. A block is released at A and slides without friction until it reaches point B. (a) How fast is it moving at B, the bottom of the quadrant? (b) The horizontal part is not smooth. If the block comes to rest 3.0 m from B, what is the coefficient of kinetic friction?

\( \square \) Conservation of energy yields the equation \( \frac{1}{2} mv_A^2 + mgh_A = mgh_A \), so that \( v_A = \sqrt{2g(h_A - h_B)} \). With \( h_A - h_B = 1.0 \text{ m} \), we obtain \( v_A = \sqrt{2(9.8)(1.0)} = 4.43 \text{ m/s} \).

(b) If the block slides distance \( d \) in coming to rest, the work-energy theorem gives \( W_r = -\mu_s mgd = -\frac{1}{2} mv_A^2 \). Then \( \mu_s = \frac{(\frac{1}{2} mv_A^2)}{(mgd)} = \frac{mgh_A - h_B}{(mgd)} = (h_A - h_B)/d \). With \( d = 3.0 \text{ m} \), \( \mu_s = \frac{1}{3} = 0.333 \).

7.74 Figure 7.14 shows the plan for a proposed roller coaster track. Each car will start from rest at point A and will roll with negligible friction. It is important that there be at least some small positive normal force (that is, a push) exerted by the track on the car at all points; otherwise the car would leave the track. What is the minimum safe value for the radius of curvature at point B?

\( \square \) We use \( v_A \) and \( v_B \) to denote the speeds of the car at points A and B, and we use \( h_A \) and \( h_B \) to denote the elevations. If the car has mass \( m \), the equation expressing energy conservation is \( \frac{1}{2} mv_B^2 + mgh_B = \frac{1}{2} mv_A^2 + mgh_A \).