Disclaimer: There are too many types of problems that we have covered this quarter for this review sheet to be representative of them. This is just a sparse sampling of what you are responsible to have learned for the exam.

1. Two forces \( \vec{F}_1 \) and \( \vec{F}_2 \) act along the sides of an equilateral triangle as shown. Point \( O \) is the intersection of the altitudes of the triangle. Find a third force \( \vec{F}_3 \) to be applied at \( B \) and along \( BC \) that will make zero total torque about point \( O \).

Call the distance \( OB \) "\( d \)"

\[
-F_1(\sin 30^\circ) - F_2(\sin 30^\circ) + F_3(\sin 30^\circ) = 0
\]

\[
F_3 = F_1 + F_2
\]

2. Write an expression that describes the pressure variation as a function of position and time for a sinusoidal sound wave in air. Assume \( \lambda = 0.1 \) m and \( \Delta P_{\text{max}} = 0.2 \) N/m².

\[
\Delta P = \Delta P_{\text{max}} \sin \left( \frac{2 \pi x}{\lambda} - wt \right)
\]

\[
k = \frac{2 \pi}{\lambda} = 62.83 \text{ m}^{-1}
\]

\[
\Delta P = (0.2 \text{ N/m}^2) \sin \left( 62.83 \text{ m}^{-1} \cdot \frac{21551 \text{ rad}}{5} \right)
\]

\[
u = \frac{343 \text{ m/s}}{62.83} = \frac{\omega}{k} = \frac{\omega}{62.83}
\]

\[
\omega = 21551 \frac{\text{rad}}{5}
\]

3. If \( x(t) = 7t^2 - 2t + 1 \) gives the position of an electron in meters after \( t \) seconds, find \( v_x(2s) \) and \( a_x(2s) \).

\[v_x(t) = 21t^2 - 2\]

\[a_x(t) = 42t\]

\[v_x(2s) = 82 \text{ m/s}\]

\[a_x(2s) = 84 \text{ m/s}^2\]

4. An 8.5 kg object passes through the origin with a velocity of 42 m/s parallel to the \( x \) axis. It experiences a constant 19 N force in the direction of the positive \( y \) axis. Calculate the velocity and the position of the object 15 s after it passes the origin.

\[
x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2
\]

\[
= 0 + 42(15) + 0 = 630 \text{ m} = x
\]

\[a_x = 0\]

\[
y = y_0 + v_{y0}t + \frac{1}{2}a_yt^2
\]

\[
= 0 + 0 + \frac{1}{2}(2.235)(15)^2 = 251.44 \text{ m} = y
\]

\[
v_x = v_{x0} + a_xt
\]

\[
= 42 + 0 \cdot t = 42 \text{ m/s} = v_x
\]

\[
v_y = v_{y0} + a_yt
\]

\[
= 0 + (2.23)(15) = 33.45 \text{ m/s} = v_y
\]
5. A simple pendulum with a length of 2.23 m and a mass of 6.74 kg is given an initial speed of 2.06 m/s at its equilibrium position. Assume that it undergoes simple harmonic motion. Determine its period, total energy, and its maximum angular displacement.

\[ \omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.81 \text{ m/s}^2}{2.23 \text{ m}}} = 2.097 \text{ rad/s} \]

\[ T = \frac{2\pi}{\omega} = 2.996 \text{ s} \]

\[ \text{Total energy} = \frac{1}{2} mv^2 = 14.3 \text{ J} \]

At max disp. \[ mg\ell (1 - \cos \theta) = 14.3 \text{ J} \]

\[ \theta = 25.44^\circ \]

6. A man of mass \( m \) clings to a rope ladder suspended below a balloon of mass \( M \). The balloon is stationary with respect to the ground.

(a) If the man begins to climb the ladder at speed \( v \) with respect to the ladder, in what direction and with what speed (with respect to the earth) will the balloon move? Your answer should involve \( M, m \) and \( v \).

No net external force \( \Rightarrow \) momentum is conserved.

Man's speed rel. to earth is \( u - v \)

\[ m(v - v) - MV = 0 \rightarrow mv = (m+M)v \]

\[ v = \frac{mv}{m+M} \]

(b) What is the state of motion when the man stops climbing?

Everything is at rest relative to the earth. (Cons. of \( \dot{\theta} \))

7. Calculate the rotational inertia of a sphere of radius 0.5 m with mass 0.56 kg about an axis that is tangent to its surface.

\[ I = \frac{2}{3} mr^2 + mr^2 = \frac{5}{3} mr^2 = 0.233 \text{ kg m}^2 \]
8 A 12 g ball of sticky clay is thrown horizontally at a 100 g wooden block initially at rest on a horizontal surface. The clay sticks to the block. After impact the block slide 7.5 m before coming to rest. If the coefficient of kinetic friction between the block and the surface is 0.650, what was the speed of the clay immediately before impact?

\[ \text{Momentums are conserved in the collision.} \]

After collision, \[ \frac{1}{2} (0.112 \text{ kg}) v_f^2 = -f d = -\mu_k N d = -(0.65)(0.112 \text{ kg})(9.81 \text{ m/s}^2)(7.5 \text{ m}) \]

\[ \Delta K \rightarrow v_f = 9.78 \text{ m/s} \]

Cons of \( v_f \): \[ (0.012 \text{ kg}) v_i = (0.112 \text{ kg})(9.78 \text{ m/s}) \rightarrow v_i = 91.28 \text{ m/s} \]

9 A playground merry-go-round of radius \( R = 2 \text{ m} \) has a moment of inertia \( I = 250 \text{ kg} \cdot \text{m}^2 \) and is rotating at 10 rev/min about a frictionless vertical axle. Facing the axle, a 25 kg child hops onto the merry-go-round and manages to sit down on the edge. What is the new angular speed of the merry-go-round?

\[ L_z \text{ is conserved.} \]

\[ L_z, i = I \omega = (250 \text{ kg} \cdot \text{m}^2)(20 \pi \text{ rad/min}) \left( \frac{1 \text{ min}}{60 \text{ sec}} \right) = 261.8 \text{ kg} \cdot \text{m}^2 \cdot \text{sec} \]

Cons of \( L_z \): \[ 261.8 \text{ kg} \cdot \text{m}^2 \cdot \text{sec} = (250 + 25(2m)^2) \omega_f \rightarrow \omega_f = 0.74799 \text{ rad/sec} \]

\[ = 7.14 \text{ rev/min} \]

10 An amusement park ride consists of a rotating circular platform 8 m in diameter from which bucket seats are suspended at the end of 2.5 m chains. When the system rotates, the chains holding the seats make an angle of \( \theta = 28^\circ \) with the vertical.

(a) What is the speed of the seat relative to the ground?

\[ \text{What linear speed makes } \theta = 28^\circ? \]

\[ T \sin \theta = \frac{mg \sin \theta}{\cos \theta} = \frac{mv^2}{r} \]

\[ \rightarrow v = \sqrt{\frac{mg \tan \theta}{2.5 \sin \theta + 4}(9.81)(\sin \theta)} \]

(b) If a child of mass 40 kg sits in the 10 kg seat, what is the tension in the chain?

\[ T \cos 28^\circ = mg \]

\[ T = \frac{mg}{\cos 28^\circ} = \frac{(50 \text{ kg})(9.8)}{\cos 28^\circ} = 554.96 \text{ N} \]
A ski jumper leaves the jump with a speed of 8 m/s at an angle of 15° above the horizontal. The slope below the jump is inclined at 50° above the horizontal, and air resistance is negligible.

(a) Find the distance that the jumper lands down the slope, measured along the line from the jump to the landing point.

\[ x = 8 \cos(15°)t = 7.727t \rightarrow t = \frac{17.727}{9.81} \]

\[ y = 8 \sin(15°)t - \frac{1}{2} g t^2 \rightarrow y = 0.2679x - 0.08215x^2 \]

These two curves intersect at the solution of \(-\tan 50°x = 0.2679x - 0.08215x^2 \rightarrow x = 17.768 \text{ m}, y = -21.175 \text{ m} \rightarrow \text{distance} = 27.64 \text{ m}\)

(b) Find the jumper’s horizontal and vertical velocity components just before landing.

\[ v_x = 8 \frac{m}{s} \cos 15° = 7.727 \frac{m}{s} = v_x \]

\[ v_y = v_y - 9.81t = 8 \sin(15°) - 9.81 \left( \frac{17.768}{7.727} \right) = -20.486 \frac{m}{s} = v_y \]

A transverse wave on a string is described by the wave function \( y = (0.120 \text{ m}) \sin \left( \frac{\pi}{8} x + 4\pi t \right) \).

(a) Determine the transverse speed and acceleration of the string at \( t = 0.2 \text{ s} \) for the point on the string located at \( x = 1.6 \text{ m} \).

\[ \frac{dy}{dt} = (0.120 \text{ m}) 4\pi \cos \left( \frac{\pi}{8} x + 4\pi t \right) \rightarrow \frac{dy}{dt} = -1.508 \frac{m}{s} \]

\[ a_y = -(0.120 \text{ m}) (4\pi)^2 \sin \left( \frac{\pi}{8} x + 4\pi t \right) \rightarrow a_y = 0 \]

(b) What are the wavelength, period, and speed of propagation of the wave?

\[ \lambda = \frac{2\pi}{\frac{\pi}{8}} \rightarrow \lambda = 16 \text{ m} \text{ (if x is in m)} \]

\[ \omega = 4\pi \sqrt{\frac{2\pi}{\sqrt{\frac{\pi}{8}}} = \frac{1}{2} \text{ s} \text{ (if t is in s)} \]

\[ v = \frac{\lambda}{T} = \frac{16}{\frac{1}{2}} = 32 \frac{m}{s} \]