Physics 210 Laboratory
The Physical Pendulum

In this experiment we study the properties of a compound pendulum. In particular, a meter stick is suspended about axes at different locations along its length. The period about each axis is measured using the rotary motion sensor and the DataStudio software. Graphs of the pendulum period versus the distance of the axis from the meter stick’s center of mass and the derivative of the period (with respect to the distance) versus this same distance yield information that can permit the determination of the rotational inertia of the meter stick about its center of mass and the meter stick’s radius of gyration.

Theory

A simple pendulum consists of a small body sometimes called a “bob” attached to the end of a string the length of which is large compared to the dimensions of the bob. Moreover, the mass of the string is negligible in comparison with that of the bob. Under these conditions the mass of the bob may be regarded as concentrated at its center of mass, and the length of the pendulum is the distance to this point from the axis of suspension. When a simple pendulum swings through a small arc, it executes simple harmonic motion of period \( T \) given by the equation

\[
T = 2\pi \sqrt{\frac{l}{g}},
\]

where \( g \) is the gravitational field strength and \( l \) is the distance from the suspension point to the center of the bob.

When the dimensions of the suspended body are not negligible compared to the distance from the suspension axis to the center of mass, the pendulum is called a physical pendulum. Any object mounted on a horizontal axis so as to oscillate under the force of gravity is a physical pendulum. The expression for the period of a physical pendulum is

\[
T = 2\pi \sqrt{\frac{l}{mg\cdot h}}.
\]

where \( l \) is the rotational inertia of the pendulum about the axis of suspension, \( m \) is the pendulum's mass, and \( h \) is the distance from the suspension point to the center of mass. Figure 1 illustrates a meter stick suspended about a rotation axis \( S \).

![Figure 1. A meter stick as a physical pendulum.](image_url)
It is convenient to express $I$ in terms of $I_{CM}$, the rotational inertia of the body about an axis through its center of mass. The rotational inertia about any axis parallel to the one through the center of mass is given by the parallel axis theorem,

$$I = I_{CM} + mh^2,$$

where $h$ is the distance between the two axes.

Imagine concentrating all of the mass of the physical pendulum to a single point, and suspending it from a massless string to form a simple pendulum. If the length of the string were such that the simple pendulum has the same rotational inertia as the physical pendulum about an axis through its center of mass, then that length is called the *radius of gyration*. Specifically,

$$I_{CM} = mR_0^2.$$

For any regular body (such as a disk, sphere, or rod) $R_0$ can be computed with an appropriate formula (see your text); for an irregular body, it must be determined experimentally. Substituting Equations (3) and (4) into Equation (2) yields the period in terms of only the geometry of the pendulum:

$$T = 2\pi \sqrt{\frac{R_0^2 + h^2}{gh}}.$$

The radius of gyration is equal to the distance $h$ when the period of oscillation is a minimum. All of our experimental results are predicated on this fact. The proof is left as an exercise for you.

Equation (5) can be rewritten such that the quantity $T^2h$ is linear in $h^2$:

$$T^2h = \left(\frac{4\pi^2}{g}\right)h^2 + \frac{4\pi^2R_0^2}{g}.$$

**Procedure**

The meter stick has holes drilled every 2.00 cm along its length. Beginning 2.00 cm from one end, attach the meter stick to the rotation shaft of the rotary motion sensor. After measuring the period of oscillation (the method is described below), remove the meter stick and reattach it at 4.00 cm from the end. Repeat this procedure every 2.00 cm to (and including) a distance of 42.0 cm from the end of the meter stick.

To measure the period of oscillation of the meter stick:

1. Load the Lab9Pendulum.ds file in DataStudio.
2. Displace the meter stick through a small arc and release it so that it swings back and forth.
3. Click the Start button. The angular position of the sensor will be recorded for 12.0 seconds.
4. To obtain an accurate measurement of the period of oscillation, move the crosshairs to a convenient point on the graph such as a peak or where the curve crosses the horizontal axis. Note this time $t_1$, and record it in a spreadsheet. Next, use the crosshairs to find the time $t_f$ after the pendulum has swung through five complete periods. Use these times to calculate the period of oscillation.
Analysis

1. Using Equation (4) and the formula for the rotational inertia of a thin rod, determine the theoretical radius of gyration of the meter stick.
2. Beginning with Equation (5), prove that the distance between the axis of rotation and center of mass \( (h) \) equals the radius of gyration when the period of oscillation is minimum. Mathematically, solve for \( h_{\text{min}} \):
   \[
   \left. \frac{dT}{dh} \right|_{h_{\text{min}}} = 0.
   \]
3. Construct a spreadsheet with the collected data as shown in the figure.

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4. Plot \( T \) vs. \( h \), and locate \( h_{\text{min}} \) (the \( h \) value where the period is shortest).
5. For a more accurate measure of \( R_0 \), plot \( T^2 h \) vs. \( h^2 \), and perform the LINEST analysis on the data. Using the slope of the line, determine \( g \) and compare it to the commonly accepted value.
6. Using the value of \( g \) calculated above, compute \( R_0 \) from the y-intercept.
7. Compare the three values of \( R_0 \) that you’ve found. Specifically, find the percent difference between the theoretical and calculated values (found in steps 1 and 5).
8. The formula for the rotational inertia of a uniformly dense bar of length \( l \) and width \( w \) is
   \[
   I_{CM} = \frac{1}{12} m(l^2 + w^2).
   \]
   Estimate the error introduced in the radius of gyration by neglecting the width of the meter stick in step 1.
9. The calculation of \( I_{CM} \) above neglected the effect of holes drilled in the meter stick. How would you estimate the error introduced by neglecting the holes?

Report

Hand in your spreadsheet, and carefully written answers to the questions posed in the analysis section.